DEMAND ELASTICITY AND MERGER PROFITABILITY

A Thesis Submitted to the
College of Graduate Studies and Research
in Partial Fulfillment of the Requirements for the
Degree of Master of Arts in the
Department of Economics
University of Saskatchewan
Saskatoon

By Yajun Wang

PERMISSION TO USE

In presenting this thesis in partial fulfilment of the requirements for a Postgraduate degree from the University of Saskatchewan, I agree that the Libraries of this University may make it freely available for inspection. I further agree that permission for copying of this thesis in any manner, in whole or in part, for scholarly purposes may be granted by the professor or professors who supervised my thesis work or, in their absence, by the Head of the Department or the Dean of the College in which my thesis work was done. It is understood that any copying or publication or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission. It is also understood that due recognition shall be given to me and to the University of Saskatchewan in any scholarly use which may be made of any material in my research project.

Request for permission to copy or to make other use of material in this thesis in whole or part should be addressed to:

Head of the Department of Economics
University of Saskatchewan
9 Campus Drive
Saskatoon, Saskatchewan S7N 5A5

ABSTRACT

This thesis is an extension of a recent study into the relationship between merger size and profitability. It studies a class of Cournot oligopoly with linear cost and quadratic demand. Its focus is to analyze how a merger's profitability is affected by its size and by the demand elasticity. Such results have not yet been reported in previous studies, perhaps due to the complexity of the equilibrium equation involved. It shows an increase in the demand elasticity also raises a merger's profitability. Consequently, an increase in the demand elasticity reduces merged members' critical combined per-merger market share for the merger to be profit enhancing. Comparing with 80% minimum market share requirement for a profitable merger in Salant, Switzer, and Reynolds (1983), a greater market share is needed when the demand function is concave (demand is relatively inelastic), while a smaller market share may still be profitable when the demand function is convex (demand is relatively elastic). In our model, mergers are generally detrimental to public interests by increasing market price and reducing output. However, the merger will be less harmful when the goods are very inelastic.

Key Words: Cournot oligopoly, demand elasticity, convexity, concavity.

ACKNOWLEDGEMENTS

I could not write a single page of this thesis if there were not those who continuously offering help and support to me. I would like to take this opportunity to thank all of them.

Most of all, thanks for my supervisor Professor Jingang Zhao who offers me this interesting topic, where I could exercise my knowledge and wits. Without his guidance, I would have been lost in the tedious mathematical proofs and calculations. I also thank my thesis committee members, Professor Murray Fulton, Eric Howe and Nazmi Sari, for their invaluable suggestions and corrections. They challenged my wisdom and helped me to find my weakest link in Economics.

I especially want to thank Professor Joel Bruneau, for his insightful discussion, always opening for questions and offering help. With his assistance in providing many cherishable intuitions for the results of my thesis, I have a clearer picture of the whole analysis. Moreover, I would also thank Professor Mobinul Huq, for his considering for students' life and always offers time for students' questions. In short, I want to thank all of the professors who offer me classes, and those who help me overcome the difficulties.

I would also take this opportunity to thank all my friends who supported me over the past three years, especially Andrew Koeman, who helped me a lot in the analysis part in this thesis.

Finally, I want to express my sincere gratitude to my beloved parents, Yonghui Wang and Xiaoqin Zhang, who always stand beside me no matter what happens. Without their love and encouragement, I would not be here now and pursue a career that I really want to do in the rest of my life.

Yajun (Francesca) Wang

June 27, 2005

To my parents

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Since the 1980s, there has been a burgeoning wave of mergers resulting in large part from globalization and free trade. However, few firms conspire to be a monopoly, even though it is always profitable, since it always attracts government regulation and arouses consumer anger, such as in the case of Microsoft. On the other hand, oligopolies can more easily avoid constant attention of the regulators, while simultaneously earning profits greater than those possible in a more competitive market. With economic globalization, the growth of oligopoly merging poses problems both for economic theory and for economic policy.

In an oligopoly model, what is the benchmark for the merger to be profitable, which determines the likelihood of the merger, and what is social welfare effect, which is vital to the policy making, become the main concern to the economists in the industrial organization. A wide range of economic literature has been devoted to providing related analysis on the effect of the merger size on the profitability.

In the earlier industrial organization literature, it is generally presumed that horizontal merging is always profit enhancing to the coalition members in the Cournot case even with constant returns to scale, since they always have the choice of reproducing the same amount of products as pre-merger level. This assumption was challenged by Salant, Switzer and Reynolds (1983) (SSR model hereafter). The model they employed uses constant marginal cost and linear demand. They showed that given the outputs of the other players, the merged firms would have an incentive to alter its production, which means the output and market prices are endogenously determined. Thus their pre-merger production level will not be an equilibrium following the merger. Furthermore, they developed the sufficient condition for the merger to be profitable, which is 80% pre-merger market share for its existing coalition members. However, the model they use is specifically simplified, and the result will not be significantly meaningful when applied to other models.

When Farrell and Shapiro (1990) analyzed horizontal mergers in a Cournot oligopoly with general demand and general costs, they found that any merger not creating synergies raises price. Furthermore, on the condition that a merger's initial joint market share does not exceed the weighted sum of the outsider's total market share in a linear Cournot model, they show that if the merger is profitable and raises the market price, it would raise welfare as well. This conclusion could be interpreted as the alternative to Levin (1990)'s finding, which shows any profitable merger whose combined pre-merger market share makes up no more than 50 percent of the total market output will raise welfare, and is the so called 50% benchmark (FSL hereafter).

However, this 50 percent rule could not be generalized to any Cournot models. Recently, work by Heubeck, Smythe, and Zhao (2005) studied a linear Cournot model with asymmetric cost. They find the set of profitable, welfare-enhancing mergers is much larger than the set defined by the 50% rule when using this more restrictive model. This result is counter-intuitive, because the 50% rule is derived from FSL, a much more general model. Furthermore, they also derive the conditions for the merger to be profitable, including the critical number of the merging firms and their combined pre-merger market share.

As the above mentioned, many economists have long observed that a merger may reduce the joint profits of the participating firms. However, a coalition might not always end up losing even if it does not occupy large market share, say 80% in the SSR model. The merger may be profitable in other models, as analyzed by Deneckere and Davidson (1985). Focusing on the case of firms that produce differentiated products with Bertrand competition, they demonstrate that mergers of any size are beneficial, and large mergers are more profitable than smaller ones. The difference in those two opposite results lies in the fact that reaction functions are typically upward sloping in price games, but downward sloping in quantity games; with price as the strategic variable, mergers will be profitable all the times.

While the previous articles together bring a more coherent understanding of how the merger size affects a coalition's profitability, the model specified in this thesis has never been studied. The model is actually an extension of a SSR model, and the only difference between these two models is: instead of linear demand, we assume a nonlinear quadratic demand function, with the demand elasticity factor d determining the convexity or concavity of the demand function. In general, with the increase in the demand elasticity, producers are inclined to produce more, and they are more profitable in the SSR model over a large range of d, the exception being that goods are relatively inelastic under a convex demand model[1]. The welfare comparison is straightforward; the social welfare is at the maximum when the demand function is convex and the minimum when the demand function is concave.

By shifting the focus from the SSR model to the pre-merger and post-merger equilibria, we demonstrate that the merged firms will contract their total output relative to the pre-merger level, while other firms will expand their output in response to the merging. Furthermore, the total output in the market is reduced after the merger, which in turn pushes up the price in the industry, regardless of whether the demand function is convex or concave. Thus, mergers are always detrimental to the consumer surplus and social welfare in this specified model. However, it does not imply that the demand elasticity has no effect on the suppliers' output

[1] When $-4\frac{(n+1)^2}{(n+2)^3\omega} < d < 0$ in the non-linear demand model, producers are more profitable than those in the SSR model.

decisions. In fact, the less elastic the goods, the more likely the post-merger output level will be close to their pre-merger level, with the same pattern exhibited in the market price as well. In this case, the merger will be less harmful.

Following the analysis of the effect of demand elasticity on output and price, we develop the sufficient conditions for a merger to be profit enhancing. Even though it is expected that the concavity or convexity of the demand function and the merger's profitability are related, the results are still appealing. Our analysis shows that when the demand function is convex ($d<0$), the greater absolute value of d, the lower the combined pre-merger market share is required for the merger to be profit enhancing. Contrary to this, when firms face a concave demand function ($d>0$), the greater the value of d, the higher the coalition's market share required. Moreover, the relationship between demand elasticity and the coalition's profitability is explored. It shows, for a profitable merger, an increase in demand elasticity raises a merger's profitability.

In summary, the structure of my thesis is organized as follows: After formally specifying the model in Chapter 2, we introduce the pre-merger equilibrium in Chapter 3, and compare it with the SSR model. Chapter 4 examines how a merger's profitability is influenced by its size, and how elasticity of demand produces effects on the profitable merger's critical combined pre-merger market share and profitability. Finally, Chapter 5 offers conclusions and suggestions for future work.

CHAPTER 2

PROBLEM DESCRIPTION

This study focuses on a horizontal merger in a symmetric Cournot Model with linear cost, and with quadratic demand, affected by a demand elasticity factor denoted by d. The purpose of my analysis is to find out how the demand elasticity factor makes our result different from the SSR model, and affects the conditions required for the merger to be profitable and welfare enhancing; as well as what the merger size benchmark is for the merger to be profitable under this model.

2.1 Hypothesis and Model Description

Consider n (n>1) firms competing as Cournot players. Firm i chooses its production $q_i \geq 0$. Let $Q = \sum_{i=1}^{n} q_i$ be the total production. As in previous studies, we assume that a unique Cournot equilibrium always exists. Each firm faces an inverse non-linear quadratic demand function $P(Q) = a - Q - \frac{d}{2}Q^2$.

Even though different marginal costs provide additional incentives to a merger, in order to focus on the effect of the shape of the demand function on the profitability, as in the SSR model, we make the following assumption: each firm operates at a constant marginal and average cost of c, thus $Ci(q) = cq_i$, where $(a,c) \in R_{++}$ with $a > c$; the demand curve is downward sloping, hence $\frac{\partial P}{\partial Q} < 0$ [2]; and d is a demand elasticity factor, $\frac{\partial^2 P}{\partial Q^2} = -d$ where the demand function is concave when d>0, and convex when d<0.

Consider m ($2 \le m \le n$) firms form a merger denoted as M， $M = \{1,...,m\}$, then for each coalition member i $(i \in M)$, it produces $q_M = \sum_{i=1}^{m} q_i$; and for each non-coalition member j $(j \notin M)$, it produces q_j. For convenience, as in the SSR model, we will refer to the subset of firms that will participate in the merger as "insiders", and other firms that will continue to behave independently after the merger as "outsiders".

The main purpose of this analysis is to find out the effects of the demand elasticity on the critical profitable merger size and on the coalition's profitability. Thus, before we derive the pre-merger and post-merger equilibria, it is necessary to examine the demand elasticity factor d and demand elasticity first.

2.2 The Feasible Range of Demand Elasticity Factor "d"

This subsection is designed to illustrate the restrictions imposed on the demand elasticity factor from our specified model, and to determine its feasible range. Notice that my results include the SSR model as a special case when d=0.

Lemma 1 $d > \max\left\{-\frac{1}{2a}, -\frac{1}{(n+\frac{1}{2})q}\right\}$

Lemma 1 is derived from the following 3 restrictions:

(1) The demand curve is downward sloping or $\frac{\partial P}{\partial Q} = -1 - dQ < 0$

[2] $\frac{\partial P}{\partial Q} = -1 - dQ < 0$, it holds automatically when d>0; when $d < 0$, $d > -\frac{1}{Q}$, this condition holds if the Second Order Condition for profit maximization holds as shown in Proof of Proposition 1 in appendix.

(2) For profit maximization, the second order condition requires: $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$,

or $d > -\frac{2}{(2n+1)q}$ (as shown in Proof of Proposition 1 in appendix)

(3) The demand function could be concave or convex, depending on d>0 or d<0.

$$P = a - Q - \frac{d}{2}Q^2 = -\frac{d}{2}(Q + \frac{1}{d})^2 + (\frac{1}{2d} + a)$$

- Concave demand function: The maximum point $(-\frac{1}{d}, \frac{1}{2d} + a)$ is located in quadrant II, or $-\frac{1}{d} < 0$ & $\frac{1}{2d} + a > 0$

- Convex demand function: The minimum point $(-\frac{1}{d}, \frac{1}{2d} + a)$ is located in quadrant IV, or $-\frac{1}{d} > 0$ & $\frac{1}{2d} + a < 0$

In summary, When d>0, the above three conditions automatically hold.

When d<0,
$$\left\{\begin{array}{ll} d > -\frac{1}{Q} = -\frac{1}{nq} & (1) \\ d > -\frac{2}{(2n+1)q} = -\frac{1}{(n+1/2)q} & (2) \\ d > -\frac{1}{2a} & (3) \end{array}\right\} \Rightarrow d > \max\left\{-\frac{1}{2a}, -\frac{1}{(n+\frac{1}{2})q}\right\}$$

It is obvious from the above analysis, that d could be indefinitely large when it is positive. However, when d is negative, it is close to “0”. Alternatively, it could be interpreted as the convex demand function tends to be linear, and the concave demand function could be curved to any degree.

2.3 The Demand Elasticity "ε" and the Demand Elasticity factor "d"

Theoretically, the greater the demand elasticity, the more responsive the demand is to the changes in the price. The parameter "d" has direct effects on the demand elasticity. In order to explore how the merger's profitability is affected by the demand elasticity, it is necessary for us to explore the relationship between the demand elasticity "ε" and the demand elasticity factor "d".

Lemma 2 A smaller "d" implies the good is more elastic, or has a bigger absolute value of "ε", assuming that P/Q remains unchanged.

Solve for Q from the demand function: $P(Q) = a - Q - \frac{d}{2}Q^2 \quad \Rightarrow \quad Q = \frac{1}{2d}(-2 + 2\sqrt{1 + 2d(a-P)}$,

as long as $d \neq 0$ [3]

Then, the demand elasticity:

$$\varepsilon = \frac{\partial Q}{\partial P}\frac{\overline{P}}{\overline{Q}} = -\left(1 + 2d(a-P)\right)^{-\frac{1}{2}}\frac{\overline{P}}{\overline{Q}}$$

Since $\frac{\partial|\varepsilon|}{\partial d} = -(a-P)\left[1 + 2d(a-P)\right]^{-\frac{3}{2}} < 0$

$\Rightarrow |\varepsilon|$ is decreasing in d, regardless of whether "d" is positive or negative.

Keep in mind that such negative relationship between demand elasticity and the demand elasticity factor "d" assumes that small changes in d have no effects on P/Q. We are in a position to explore the pre-merger and post-merger equilibrium.

[3] The other solution $Q = \frac{1}{2d}(-2 - 2\sqrt{1 + 2d(a-P)}$ could be eliminated from our analysis, because when d>0, it is negative, and when d<0, the demand function is upward sloping, which is contradictory with our previous assumption that $\frac{\partial P}{\partial Q} < 0$.

CHAPTER 3

PRE-MERGER AND POST-MERGER EQUILIBRIUM

This chapter presents equilibrium outputs, profits and price, which is crucial to further analysis on the merger's profitability. To gain a more accurate picture of our proposed model, we compare it with the SSR model. A comparison between the pre-merger and post-merger equilibrium regarding the output and price is offered as well.

3.1 Pre-merger Equilibrium

The following proposition characterizes the market in equilibrium before the coalition forms, including the merged members' individual output, profit, and their combined profit, as well as the market output level, industry price, producer surplus and consumer surplus.

Proposition 1 *Let the pre-merger equilibrium individual firm's output and profit be* q_i^0, π_i^0 *respectively. Let* π_M^0 *be the merged members' combined pre-merger profits, and let the total industry output, price, profit and consumer surplus be* Q^0, P^0, π^0, CS^0 *respectively, then:*

$$q_i^0 = \frac{1}{dn(n+2)}\left[-n - 1 + \sqrt{(n+1)^2 + 2dn(a-c)(n+2)}\right]$$

Notice that the other solution $q_i^0 = \frac{1}{dn(n+2)}\left[-n - 1 - \sqrt{(n+1)^2 + 2dn(a-c)(n+2)}\right]$ was eliminated from our analysis, because of the downward sloping demand assumption.

$$Q^0 = \frac{1}{d(n+2)}\left[-n - 1 + 2\sqrt{(n+1)^2 + 2dn(a-c)(n+2)}\right]$$

$$P^0 = \frac{d(n+2)(cn+2a)+(n+1)-\sqrt{(n+1)^2 + 2nd(n+2)(a-c)}}{d(n+2)^2}$$

$$\pi_i^0 = (P^0 - c)q_i^0$$

$$\pi_M^0 = m(P^0 - c)q_i^0$$

$$\pi^0 = (P^0 - c)Q^0$$

$$CS^0 = \int_0^{Q^0} (a - Q - \frac{d}{2}Q^2)dQ - P^0 \times Q^0$$

Notice that:

- When d=0, these results become identical to the SSR model.
- Second order condition requires that $d > -\frac{1}{(n+\frac{1}{2})q^0}$

All proofs of proposition 1 are in the Appendix, including the detailed formula for profits and consumer surplus.

3.2 Comparing with the SSR Model

The only difference between the model we specified and the SSR model is the quadratic inverse demand function $P(Q) = a - Q - \frac{d}{2}Q^2$ compared with $P(Q) = a - Q$ in the SSR model. The questions remain: Does concavity (convexity) in demand increase or decrease equilibrium price, individual supply and profit? What about the effect on the consumer surplus and welfare? Proposition 2 sets about answering these questions. To simplify the analysis further, we compare these two models in per-merger equilibrium.[4] Before presenting this proposition, we first introduce some results from the SSR model: $q_{iL}^0 = \frac{a-c}{n+1}$, $\pi_{iL}^0 = \frac{(a-c)^2}{(n+1)^2}$ and $P_L^0 = \frac{a+cn}{n+1}$, where q_{iL}^0, π_{iL}^0 are each producer's output and profit respectively, and P_L^0 is the market price in the pre-merger equilibrium in the linear demand model.

Proposition 2 *Let* q_i^0 & q_{iL}^0, P^0 & P_L^0, π_i^0 & π_{iL}^0 *be the Cournot pre-merger equilibrium individual firm's outputs, market price and individual firm's profit for quadratic and linear demand models respectively. Then:*

(1) $q_i^0 < q_{iL}^0$, if d>0

$q_i^0 > q_{iL}^0$, if d<0

(2) $P^0 > P_L^0$, if d>0

$P^0 < P_L^0$, if d<0

(2) $\pi_i^0 < \pi_{iL}^0$, if $d < -4\frac{(n+1)^2}{(n+2)^3\omega}$ or $d > 0$

$\pi_i^0 > \pi_{iL}^0$, if $-4\frac{(n+1)^2}{(n+2)^3\omega} < d < 0$

$\pi_i^0 = \pi_{iL}^0$, if $d = -4\frac{(n+1)^2}{(n+2)^3\omega}$ or $d = 0$

The above results indicate that when individual firms face a concave demand function, they will produce fewer outputs, and the product market price will be higher compared with the linear demand model. The result will be the opposite when we apply this to a convex demand

[4] Notice that the result will be the same when we derive from the post-merger equilibrium, except $(n-m+1)$ is substituted for n.

function model. In general, producers in the SSR model earn more profit, except when the demand is relatively inelastic under a convex demand function model. Proposition 2 may be illustrated by the below example.

Example 3.1 Consider there are n=100 firms in the market and each operates with constant marginal and average cost: c=40, and the demand function is $P = 100 - Q - \frac{d}{2}Q^2$, each supplier produces q_i^0, gains profit π_i^0, and the market price is P^0. When the demand function is $P = 100 - Q$, each supplies q_{iL}^0, gains profit π_{iL}^0, and the market price is P_L^0.

Each individual firm's output difference between these two models would be

$$q_i^0 - q_{iL}^0 = -\frac{1}{1030200}\frac{10201 - 101\sqrt{10201 + 1224000\,d\,)} + 612000\,d}{d}$$

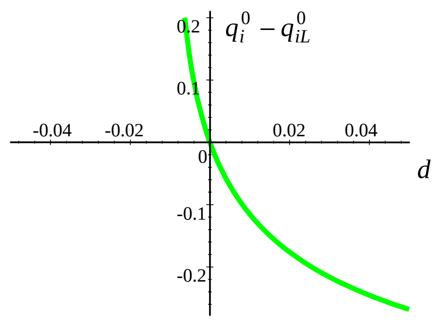
Then:

$q_i^0 < q_{iL}^0$, if $d > 0$

$q_i^0 > q_{iL}^0$, if $d < 0$

Note: the Vertical axis is the product difference.

Figure 3.1: Output Comparison with the SSR Model



The industry price difference between these two models would be:

$$P^0 - P_L^0 = \frac{1}{1050804}\frac{10201 - 101\sqrt{10201 + 1224000d} + 612000d}{d}$$

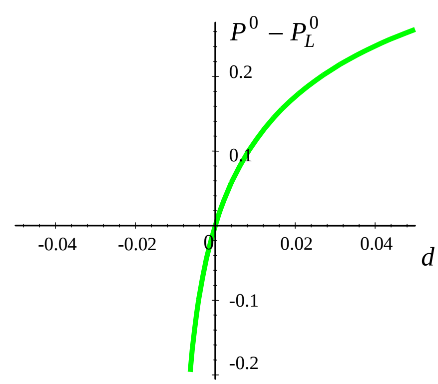
Then

$P^0 > P_L^0$, if $d > 0$

$P^0 < P_L^0$, if $d < 0$

Note: the Vertical axis is the price difference.

Figure 3.2 Price Comparison with the SSR Model



Each individual firm's profit difference between these two models would be:

$$\pi_i^0 - \pi_{iL}^0 = \frac{1}{541269140400} \frac{(62430120d + 1030301)\sqrt{10201 + 1224000d} - 12548454120d - 104060401 + 191017440000d^2}{d^2}$$

Then

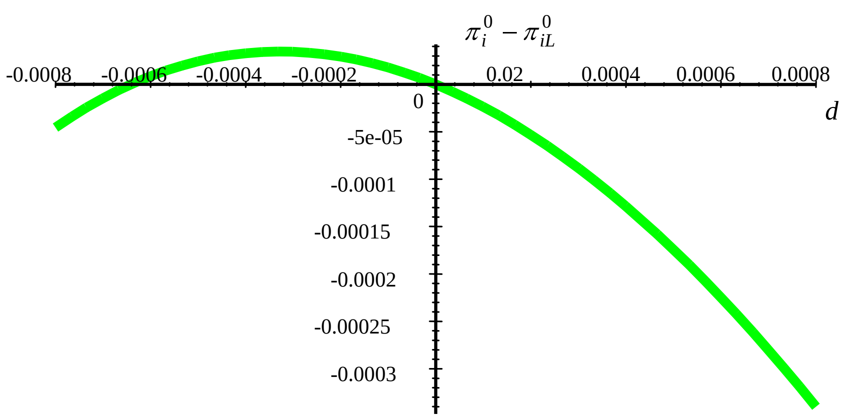
$\pi_i^0 < \pi_{iL}^0$, if $d < -\frac{10201}{15918120}$ or $d > 0$

$\pi_i^0 > \pi_{iL}^0$, if $-\frac{10201}{15918120} < d < 0$

$\pi_i^0 = \pi_{iL}^0$, if $d = -\frac{10201}{15918120}$ or $d = 0$

Figure 3.3 shows the change in profit difference with respect to the change in d.

Figure 3.3 Profit Comparison with the SSR Model



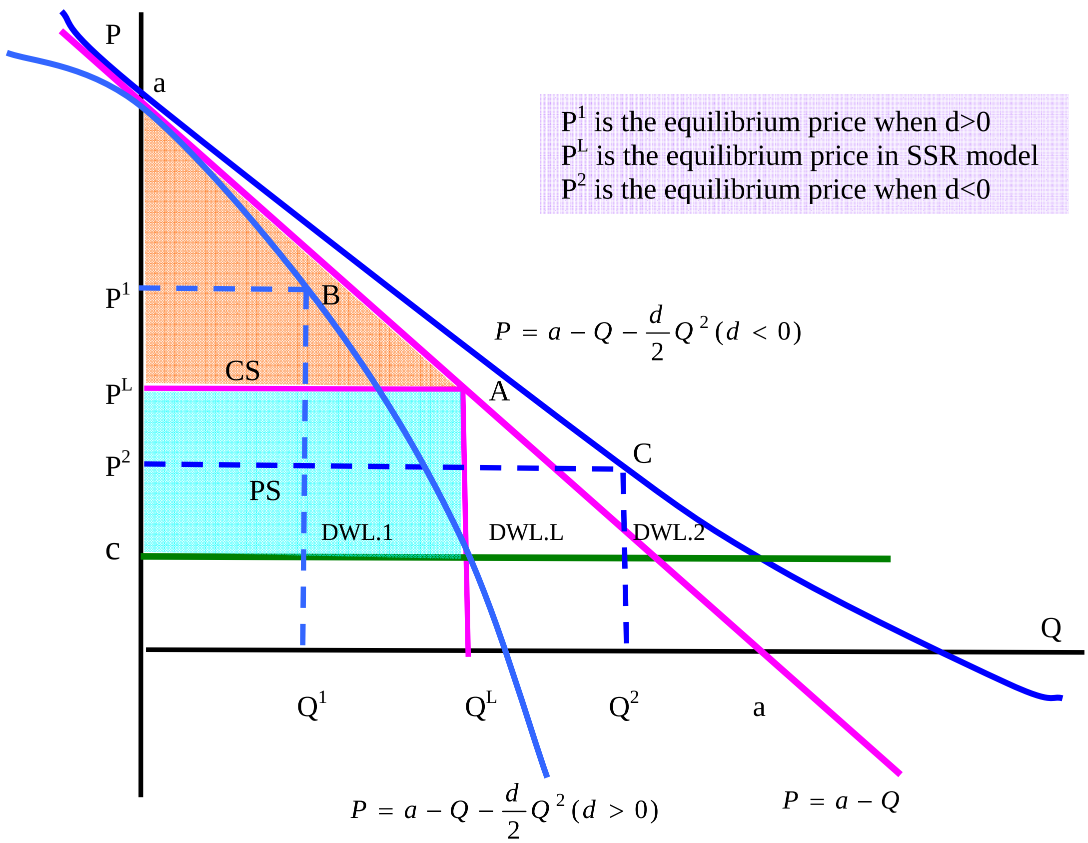
Social welfare is always a concern to most economists and policy makers; corollary 1 compares the consumer surplus and welfare effects between SSR and our model.

Corollary 1 Let $CS_{d>0}^0$ and $W_{d>0}^0$ be the Cournot pre-merger equilibrium consumer surplus and welfare when the demand function is concave, and let $CS_{d<0}^0$ and $W_{d<0}^0$ be the consumer surplus and welfare when the demand function is convex, and let CS_L^0 and W_L^0 be the consumer surplus and welfare in the SSR model, then:

(1.1) $CS^0_{d>0} < CS^0_L < CS^0_{d<0}$

(1.2) $W^0_{d>0} < W^0_L < W^0_{d<0}$

Figure 3.4 Comparisons between Quadratic Demand Model and the SSR Model



In Figure 3.4

Point A (P^L, Q^L) is the equilibrium in the SSR model.

Point B (P^1, Q^1) is the equilibrium when the demand function is concave.

Point C (P^2, Q^2) is the equilibrium when the demand function is convex.

The comparison between our quadratic non-linear model and the SSR model can be illustrated in Figure 3.4. Even though it is ambiguous to rank the industry profits among these three cases, it is quite obvious from this graph that consumers will gain due to increased output and reduced price in the market when $d<0$, and will definitely lose when $d>0$, compared with the linear demand model. As for the welfare effect, when $d<0$, since the gain to the consumer is so big, it is sufficiently large enough to preponderate over the loss to the producers (if they have a loss) and therefore welfare increases. On the contrary, when $d>0$, welfare is smaller compared with the SSR model. However, it is not clear which demand curve is the optimal one to achieve social efficiency, since the deadweight loss varies with both the output level and the demand elasticity.

A few words must be added in linking the results presented to the theoretical discussion of demand elasticity and suppliers' output production decisions. In a quantity-setting game, when the product is more elastic (d becomes smaller), producers are inclined to produce more, because a huge output expansion could only be followed by a marginal decrease in output price. As for consumers, they definitely will gain because of the lower price and higher outputs. And social welfare will increase as well, compared with a product that is less elastic.

All of the previous discussions focus on the pre-merger equilibrium. There is still the question of how the merger has an impact on this equilibrium. The next section presents the post-merger equilibrium.

3.3 Post-merger Equilibrium

Before analyzing the output, price and profitability effect of the merger, it is necessary to obtain the post-merger equilibrium. The below proposition shows the equilibrium insiders' output and profit, the outsider's output and profit, and the output of the whole industry, market price, as well as consumer surplus and producer surplus after the merger.

Proposition 3 *Let the post-merger equilibrium individual firm's output and profit be* q_i^*, π_i^*. *Let* π_M^* *be the combined merged parties' profits, and let the total industry output, price, profit and consumer surplus* be Q^*, P^*, π^* and CS^* respectively. *Then*

$$q_i^* = \frac{1}{d(n-m+1)(n-m+3)}\left[-(n-\mathrm{m}+2)+\sqrt{(n-m+2)^2+2d(a-c)(n-m+1)(n-m+3)}\right]$$

$$Q^* = \frac{1}{d(n-m+3)}\left[-(n-\mathrm{m}+2)+\sqrt{(n-m+2)^2+2d(a-c)(n-m+1)(n-m+3)}\right]$$

$$P^* = \frac{d(n-m+3)(c(n-m+1)+2a)+(n-m+2)-\sqrt{(n-m+2)^2+2d(a-c)(n-m+1)(n-m+3)}}{d(n-m+3)^2}$$

$$\pi_i^* = (P^*-c)q_i^*$$

$$\pi_M^* = (P^*-c)q_i^*$$

$$\pi^* = (P^*-c)Q^*$$

$$CS^* = \int_0^{Q^*}\left(a-Q-\frac{d}{2}Q^2\right)dQ - P^*\times Q^*$$

Notice that $d > \dfrac{-2}{\{2(n-m+1)+1\}q_i^*}$, from S.O.C.

3.4 Pre-merger and Post-merger Equilibrium Comparisons

The purpose of this subsection is to demonstrate the impact of merging on the individual firm's output decision, both for insiders and outsiders, as well as the industry output and price. Furthermore, it analyzes if the demand elasticity factor "d" has an impact on the equilibrium. It gives us a clearer picture of how the market reacts in response to the merger, and offers a foundation for further analysis on a coalition's profitability.

In order to derive the impact of "d" on total output change in the market, we impose the following restriction on the demand function.

Assumption 1 (A1)[5]: $\frac{\partial P}{\partial Q} + \frac{\partial^2 P}{\partial Q^2} q_i < 0$[6] *for all* $0 \le q_i \le Q$ *, as long as P>0*

There are two different interpretations for this assumption in the literature:

- Hahn (1962) first made this assumption, and he interpreted it as: at all possible outputs, the marginal revenue of any one producer with a given output is a diminishing function of the total output of his rivals.
- Ruffin (1971) shows the reasonableness of this assumption, and offers an alternative interpretation, which is: "at all possible outputs, the marginal revenue function facing any firm is steeper than the demand function."
- Levin (1990) mentions A1 as an important extension of the example in the SSR which assumes that $\frac{\partial^2 P}{\partial Q^2} = 0$.

The next proposition compares the pre-merger and post-merger equilibrium.

Proposition 4 *Let the pre-merger equilibrium individual firm's output, the total industry output and price be* q_i^0 *,* Q^0 *, and* P^0 *respectively. Let the post-merger equilibrium individual firm's output, the total industry output and price be* q_i^* *,* Q^* *and* P^* *respectively. Then* $q_i^* - mq_i^0$ *will be the total insiders' output change,* $q_i^* - q_i^0$ *will be each outsider firm's output change,* $Q^* - Q^0$ *will be the total industry output change, and* $P^* - P^0$ *will be the market price*

5 Levin (1990) uses this assumption to derive the total output impact of the merger in a general model, which shows that the total output in the industry will expand (contract) if the merger expands (contracts) its output level.

6 $\frac{\partial P}{\partial Q} + \frac{\partial^2 P}{\partial Q^2} q_i = -1 - d(Q+q) < 0$

This condition automatically holds when d>0, and when d is negative, $0 > d > \frac{-1}{q(n+1)}$.

difference. Let q_{iL}^* , Q_L^* *and* P_L^* *be the Cournot post-merger equilibrium coalition's combined outputs (or each outsider's output), total market outputs, and market price in the SSR model respectively. Then,*

(1) *The Merger leads to a contraction of the insiders' total output relative to the pre-merger level.*

$q_i^* < mq_i^0$, as long as $d \neq 0$

Remark: $\lim_{d \to \infty} q_i^* = mq_i^0$

$\lim_{d \to 0} q_i^* = q_{iL}^*$

(2) Each *outsider will expand their output in response to the merging.*

$q_i^* > q_i^0$, as long as $d \neq 0$

Remark: $\lim_{d \to \infty} q_i^* = q_i^0$

$\lim_{d \to 0} q_i^* = q_{iL}^*$

(3) *The Merger leads to the contraction of the total output in the market relative to the pre-merger level.*[7]

$Q^* < Q^0$, as long as $d \neq 0$

Where $Q^* = Q_M^* + Q_F^*$ and $Q^0 = Q_M^0 + Q_F^0$, for all i $(i \in M)$ & j $(j \in F)$.

Where Q_M^0 is the insiders' total outputs before the merger.

Where Q_F^0 is the outsiders' total outputs before the merger.

Where Q_M^* is the insiders' total outputs after the merger.

Where Q_F^* is the outsiders' total outputs after the merger.

Remark: $\lim_{d \to \infty} Q^* = Q^0$

[7] Levin (1990) proved proposition 4, (3) by assuming proposition 4, (1) for a general case.

$$\lim_{d \to 0} Q^* = Q_L^*$$

As for the price effect, it is obvious that the contraction of the output supply will always push up the price. The related proofs for proposition 4 are in the appendix.

Given the pre-merger output level of the outsiders, the insiders tend to reduce their post-merger outputs. On the other hand, the outsiders will expand their outputs in response to a higher price, which is one of the main effects of the merging. They become the free riders in the industry, followed by further output reduction of insiders. After all of these adjustments have taken place, coalition members may be unprofitable compared with the pre-merger level; since the gain from a higher price could not outweigh the loss from lower output, their share of the pie decreased. Not surprisingly, in order to overcome outsiders' output expansion effect, a merger needs to have at least 80% market share for it to be profitable in the SSR model.

Since in our model, the demand elasticity factor d is a crucial factor for its effect on the profitability conditions, I would like to include the graphs for positive d and negative d respectively, where the demand function is concave when d>0, and convex when d<0. Furthermore, from these diagrams, we see that the range of d required to fulfill the concave or convex demand function assumptions has been satisfied. As for the consumer surplus and welfare effect, we can achieve the conclusion without numerical proof.

Corollary 2

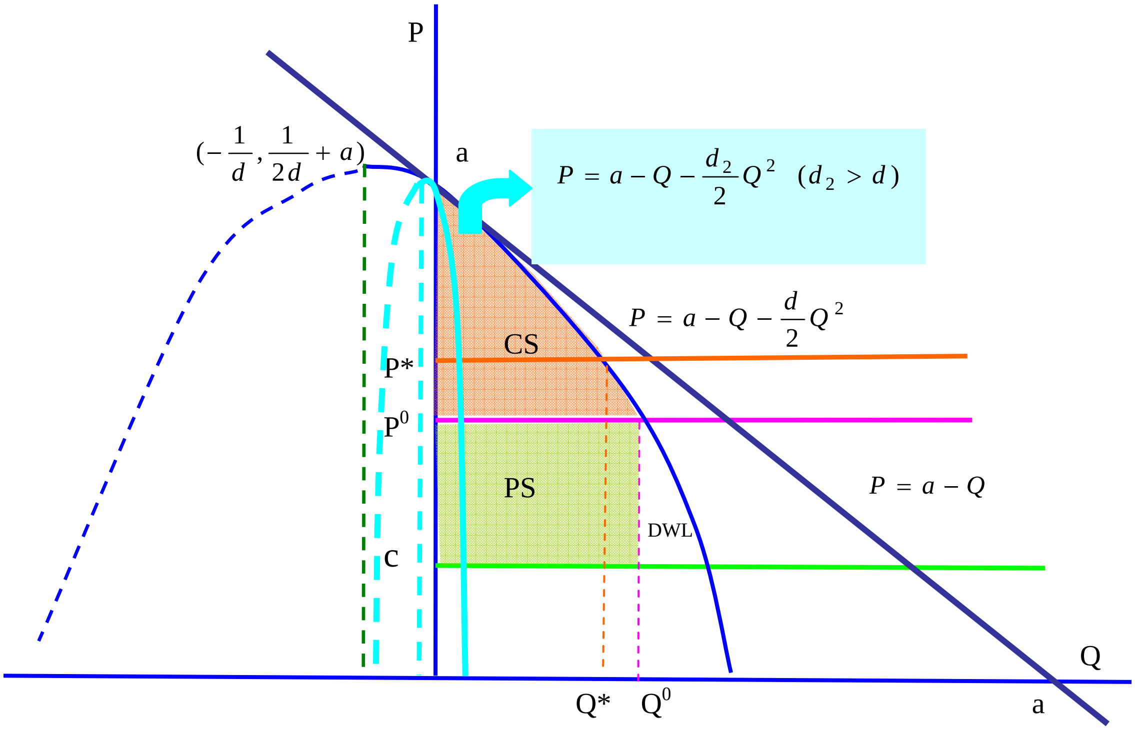
(2.1) We could apply the SSR results to our specified quadratic demand model when d is close to "0", since the demand function tends to be linear.

(2.2) $CS^0 > CS^*$**,** due to increase in price and decrease in output in the market.

(2.3) $W^0 > W^*$**,** because of the increase in the deadweight loss after the merger.

Figure 3.5 Comparisons between Pre-merger and Post-merger Equilibrium

(when d>0)

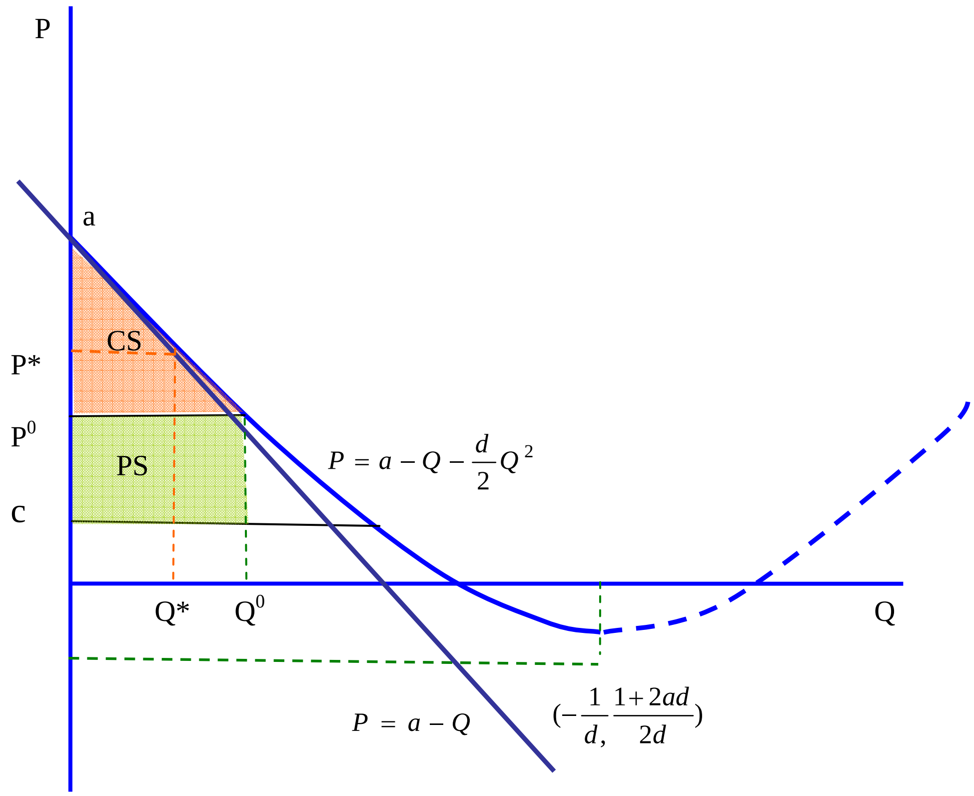


In Figure 3.5 and 3.6, the convexity or concavity of the demand function does not change the main outcomes of merging. However, when the demand elasticity factor is very large, which implies the good is perfectly inelastic (it could be represented by the inner light demand function in Figure 3.5, where $d_2 > d$), it will be difficult for the coalition to reduce the output level by a large enough factor to increase the market price. In this case, the merger will be less harmful.

The results from proposition 4 and corollary 2 are consistent with merger theory. The aim of merging is to increase profit by reducing the competition. An industry will be less competitive when there are fewer firms. The Cournot oligopoly players in this model will restrict the amount of output they produce in order to push up the price, and this is detrimental to the public interests.

Figure 3.6 Comparisons between Pre-merger and Post-merger Equilibrium

(when d<0)



CHAPTER 4

PROFITABILITY EFFECT

Only profitable mergers will exist, and are of policy makers and economists' interests. This chapter derives the sufficient conditions for a merger to be profit enhancing relative to the pre-merger equilibrium. It also explores how demand elasticity affects the merger's profitability.

4.1 Profitability Conditions

Equation 4.1 gives the merger's profit change, where π_M^* is the merger's post-merger profit, and π_M^0 is the insiders' combined pre-merger profit.

$$\pi_M^* - \pi_M^0 = 2\frac{\left\{d\omega(Y+1)(-2Y+\sqrt{\beta}+1)+Y(-Y+\sqrt{\beta})\right\}}{d^2(Y+1)^2(Y^2-1)} - 2m\frac{\left\{d\omega(X+1)(-2X+\sqrt{\alpha}+1)+X(-X+\sqrt{\alpha})\right\}}{d^2(X+1)^2(X^2-1)} \qquad \textbf{(4.1)}$$

Where $\alpha = X^2 + 2d\omega(X^2-1)$
$\beta = Y^2 + 2d\omega(Y^2-1)$

Where $\omega = a - c$

$X = n + 1$

$Y = n - m + 2$

Proposition 5 *Let* m^* *and* θ^* *be the critical merger size and the critical market share for a profitable merger, then the merger is profit enhancing for the insiders if any of the following 2 claims is met:*

(i) $m^* < m$

(ii) $\theta^* < \theta$

Where $m^* = \rho$, and ρ is a root of $\beta_0 + \beta_1 Z + \beta_2 Z^2 + \beta_3 Z^3 + \beta_4 Z^4 + \beta_5 Z^5 + \beta_6 Z^6 = 0$ [8] **(4.2)**

In order to simplify the notation further, let:

$\alpha = (1+2d\omega)n^2 + (2+4d\omega)n + 1$, which is the same as $\alpha = X^2 + 2d\omega(X^2 - 1)$ in equation 4.1.

$P_1 = d\omega(n+2) + (n+1)$,

$P_2 = (n+2)(2n+1)d\omega + (n+1)^2$

Then: $\beta_0 = -d^2\omega^2 n^2 (n+2)^6 (2d\omega + 1)$

$\beta_1 = 2n(n+2)^3 \left(d\omega(2n^2 + 9n + 9) + (n+2)^2\right)(P_1\sqrt{\alpha} - P_2)$

$\beta_2 = (n+1)(n+3)^3 (P_1\sqrt{\alpha} - P_2)^2 - 2n(n+2)^3 \left(d\omega(4n+9) + 2(n+2)\right)(P_1\sqrt{\alpha} - P_2)$

$\beta_3 = 2n(n+2)^3(2d\omega+1)(P_1\sqrt{\alpha} - P_2) - 2(2n+3)(n+3)^2(P_1\sqrt{\alpha} - P_2)^2$

$\beta_4 = 6(n+3)(n+2)(P_1\sqrt{\alpha} - P_2)^2$

$\beta_5 = -2(5+2n)(P_1\sqrt{\alpha} - P_2)^2$

$\beta_6 = (P_1\sqrt{\alpha} - P_2)^2$

Where $\theta^* = \dfrac{m^*}{n}$

[8] There are 6 solutions to this equation. However, there is only one feasible solution.

The above analysis indicates that an increase in the merger size or market share would increase a merger's profitability. We will use example 4.1 to illustrate proposition 5.

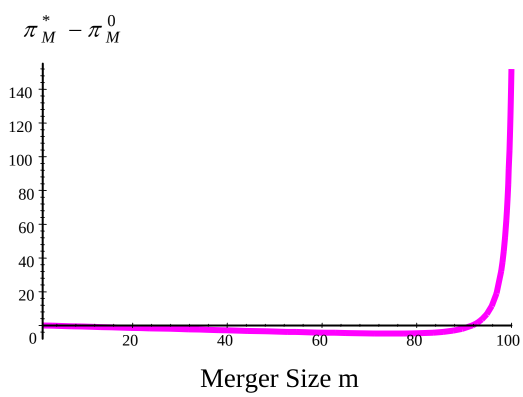
Example 4.1: Consider a market of 100 firms, where each firm operates at a constant marginal and average cost: c=40. The demand function is: $P = 100 - Q - Q^2$, where $d = 2$. In order to find the critical value of the merger size m^*, let $\pi_M^* - \pi_M^0 = 0$, or

$$0.25\frac{2.0(12462-121m)\sqrt{2.50712\times10^6 - 49164m + 241m^2} - 240(103-m)(203-2m) - 2(102-m)^2}{(103-m)^3(101-m)} \qquad \textbf{(4.3)}$$
$$-7.95253\times10^{-2}m = 0$$

Then $m_1 = 103.153 + 11.2815i, m_2 = 103.153 - 11.2815i, m_3 = 0.999999, m_4 = 91.8451, m_5 = 100.992$

$\Rightarrow m^* = 91.8451 \Rightarrow \theta^* = 91.845\%$

Figure 4.1 Merger's sizes and its Profitability



- The horizontal axis represents the merger size.
- The vertical axis represents the merger's profitability change.

Based on equation 4.3, we derive Figure 4.1. When the coalition members' combined pre-merger market share is greater than 91.845%, the merger is profitable. As noted from the above figure, when the merger is large enough to be profitable, an increase in one more coalition member will increase the merger's profit by a large amount, which could be represented by the very steep upward sloping part of the curve. In this very incompetitive market, even the free riders' output expansion could not prevent the merger from being profitable.

Even though this figure shows that under concave demand, greater market share is required for the merger to be profit enhancing compared with the linear demand model, this single example could not be used to systematically evaluate the relationship between the demand elasticity and critical merger size or critical combined pre-merger market. The next section gives us a more detailed analysis.

4.2 Demand Elasticity, Profitable Merger size and Profitable Combined Pre-merger Market Share

The aim of this subsection is to explore if a change in demand elasticity has an impact on the critical merger size and the critical market share required for a merger to be profit enhancing. With the assistance of Figure 4.2 and Figure 4.3, we could have a clearer picture of the demand elasticity's impact. Before we go further, we would like to explain the variables in these two figures.

1. Figure 4.2 is based on Table 4.1[9]. The data in this table is calculated following the procedure of Example 4.1. Where:

- n is the number of firms in the industry or the market size.
- m values are all the critical merger sizes to let the insiders' pre-merger profit π_M^0 and post-merger profit π_M^* be equal.
- m^* is the feasible critical merger size for a profitable merger.

2. Figure 4.3 is derived from Table 4.2. Where:

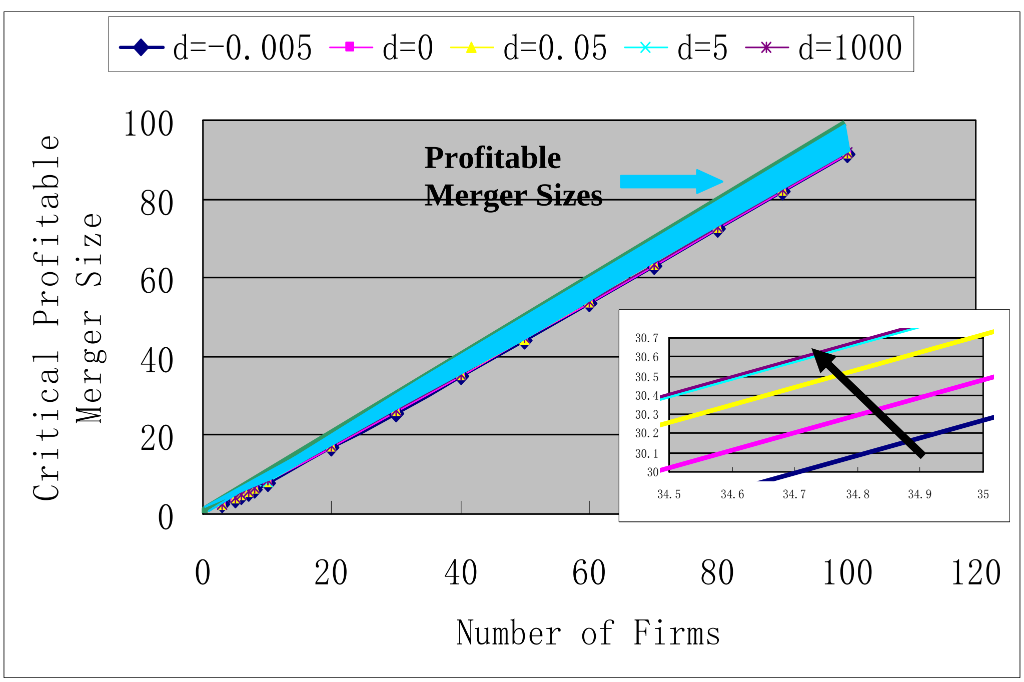
- m and n are the same as in Figure 4.2.
- m^*/n is the critical insiders' combined pre-merger market share for the merger to be profit enhancing.

[9] Table 4.1 and Table 4.2 are derived from Table A1 in the appendix.

Table 4.1: The Critical Merger Sizes for Profitable Mergers Given Different Demand Elasticity Factors and Market Sizes

n	m^*				
	d=-0.005	d=0	d=0.005	d=5	d=1000
3	2.36655	2.43845	2.53012	2.58363	2.58967
5	3.89158	4	4.13216	4.20868	4.21737
6	4.68648	4.80742	4.95299	5.03702	5.04664
7	5.49654	5.62772	5.78412	5.87417	5.88446
8	6.31887	6.45862	6.62399	6.71901	6.72985
10	7.9925	8.1459	8.32541	8.42815	8.4399
20	16.7003	16.8902	17.1067	17.2292	17.2431
30	25.7026	25.9098	26.1434	26.2747	26.2896
40	34.8596	35.0774	35.3212	35.4581	35.4735
50	44.1159	44.3411	44.592	44.7324	44.7479
60	53.4431	53.6738	53.9299	54.0729	54.0884
70	62.8241	63.059	63.3193	63.4642	63.4797
80	72.2477	72.4861	72.7497	72.8962	72.9133
90	81.7062	81.9475	82.2138	82.3617	82.3782
100	91.1939	91.4377	91.7063	91.8553	91.872

Figure 4.2: The Critical Merger Sizes for Profitable Mergers Given Different Demand Elasticity Factors and Market Sizes



The trend in Figure 4.2 shows, with the increase in the number of the firms in the market, the critical number of insiders is steadily increasing in order to let the coalition be profitable. However, the effect of the demand elasticity factor is very marginal, which makes 5 lines squeeze into one, which is represented by the lower boundary of the shaded triangle. From the magnified figure at the right hand side of the corner, we find that an increase in the demand elasticity factor d increases the critical profitable merger size requirement, but at a dramatically decreasing speed. The lowest line represents the critical profitable merger sizes when d is -0.005, which is the lowest feasible value of the demand elasticity factor, thus all merger sizes above this line will be profit enhancing.

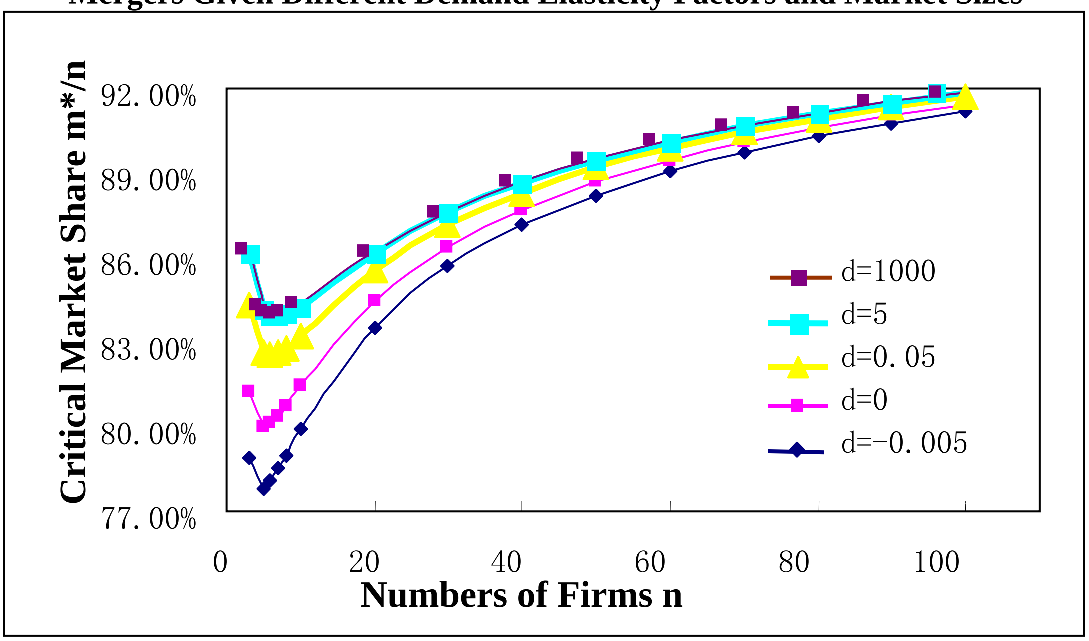
In the standardized diagram, the upper boundary of this triangle is the 45-degree line. All the regions below this line are the feasible merger sizes. The lower boundary of the triangle is the critical profitable merger sizes when d is -0.005. Thus all the merger sizes lying between these two boundaries (The shaded triangle) are profitable merger sizes.

There are many sources of economic power, such as vertical integration and the ability to operate on a global scale, and one main source is the merged members' combined pre-merger market share (market share hereafter), which is what we wish to analyze in the rest of this section. The impact of the demand elasticity on the critical profitable market share is more pronounced in Figure 4.3, compared with its influence on the critical merger size for a profitable merger. When the demand curve is linear, it needs at least 80% of the market share to form a profitable merger, which is consistent with the prediction in the SSR model; when the demand function is concave, it needs more than 80% of the market share; and when the demand function is convex, it needs at least 77.8% of the market share for the merger to be profit enhancing.

Table 4.2: The Critical Combined Pre-merger Market Shares for Profitable Mergers Given Different Demand Elasticity Factors and Market Sizes

n	m^*/n				
	d=-0.005	**d=0**	**d=2**	**d=5**	**d=1000**
3	78.89%	81.28%	84.34%	86.12%	86.32%
5	77.83%	80%	82.64%	84.17%	84.35%
6	78.11%	80.12%	82.55%	83.95%	84.11%
7	78.52%	80.40%	82.63%	83.92%	84.06%
8	78.99%	80.73%	82.80%	83.99%	84.12%
10	79.93%	81.46%	83.25%	84.28%	84.40%
20	83.50%	84.45%	85.53%	86.15%	86.22%
30	85.68%	86.37%	87.14%	87.58%	87.63%
40	87.15%	87.69%	88.30%	88.65%	88.68%
50	88.23%	88.68%	89.18%	89.46%	89.50%
60	89.07%	89.46%	89.88%	90.12%	90.15%
70	89.75%	90.08%	90.46%	90.66%	90.69%
80	90.31%	90.61%	90.94%	91.12%	91.14%
90	90.785%	91.05%	91.35%	91.51%	91.53%
100	91.19%	91.44%	91.71%	91.86%	91.87%

Figure 4.3: The Critical Combined Pre-merger Market Shares for Profitable Mergers Given Different Demand Elasticity Factors and Market Sizes



Alternatively, we could interpret the above results as: a merger facing an inelastic demand requires more market share to be profitable compared to one confronting an elastic demand market. It is also worthwhile to note that this difference tends to converge when there are more firms in the industry.

Even though the complexity of the equation prevents me from testing the effects of demand elasticity in a much larger size market with extremely large demand elasticity factor, it is still predictable that in the case of concave demand function, when the goods are perfectly inelastic in an extremely large market, only a monopoly will be profitable. As for the convex demand function market, the limitation of our specified model would not allow a very large elasticity of demand.

Now let us rule out the effect of market size, and concentrate on the demand elasticity's effects on the critical merger size and profitability. In order to simplify our analysis, we assume there are 100 firms in the industry, and it will be easier for us to derive the critical profitable merger's market share. We further assume the demand function is: $P = 100 - Q - \frac{d}{2}Q^2$, and each firm is operating at a constant marginal and average cost c=40.

Table 4.3: The Critical Merger Sizes and the Combined Pre-merger Market Shares for Profitable Mergers Given Different Demand Elasticity

d	**m value**	**m^***	**m^*/n**
-0.005	$m_1 = 91.1939, m_2 = 1.0,$ $m_3 = 110.51 - 40.5798i, m_4 = 110.51 + 40.5798i$	91.1939	91.19%
-0.004	$m_1 = 91.2747, m_2 = 0.999999,$ $m_3 = 115.166 - 53.8523i, m_4 = 115.166 + 53.8523i$	91.2747	91.27%
-0.003	$m_1 = 91.332, m_2 = 0.999999,$ $m_3 = 123.202 - 73.1373i, m_4 = 123.202 + 73.1373i$	91.332	91.33%
-0.002	$m_1 = 91.3753, m_2 = 1.0,$ $m_3 = 138.75 - 105.697i, m_4 = 138.75 + 105.697i$	91.3753	91.38%

-0.0001	$m_1 = 91.4353, m_2 = 0.999963,$ $m_3 = 588.461 - 898.34i, m_4 = 588.461 + 898.34i$	91.4353	91.44%
0	$m_1 = 91.4377, m_2 = 0.999999, m_3 = 111.562$	91.4377	91.44%
0.0001	$m_1 = 91.4402, m_2 = 1.00001,$ $m_3 = 592.65 - 905.619i, m_4 = 592.619 + 905.619i$	91.4402	91.44%
0.01	$m_1 = 91.5774, m_2 = 0.999999,$ $m_3 = 112.032 - 46.3012i, m_4 = 112.032 + 46.3012i$	91.5774	91.58%
0.03	$m_1 = 91.6677, m_2 = 0.999998,$ $m_3 = 105.978 - 26.4033i, m_4 = 105.978 + 26.4033i$	91.6677	91.67%
0.05	$m_1 = 91.7063, m_2 = 1.0,$ $m_3 = 104.886 - 21.5561i, m_4 = 104.886 + 21.5561i$	91.7063	91.71%
0.07	$m_1 = 91.729, m_2 = 1.0,$ $m_3 = 104.428 - 19.2507i, m_4 = 104.428 + 19.2507i$	91.729	91.73%
1	$m_1 = 91.8339, m_2 = 100.983, m_3 = 0.999996,$ $m_4 = 103.234 - 11.8906i, m_5 = 103.234 + 11.8906i$	91.8339	91.83%
2	$m_1 = 91.8451, m_2 = 0.999999, m_3 = 103.153 - 11.2815i,$ $m_4 = 103.153 + 11.2815i, m_5 = 100.992$	91.8451	91.85%
5	$m_1 = 91.8553, m_2 = 1.0, m_3 = 103.087 + 10.7655i,$ $m_4 = 103.087 - 10.7655i, m_5 = 100.997$	91.8553	91.8553%
20	$m_1 = 91.8642, m_2 = 100.999, m_3 = 1.0,$ $m_4 = 103.034 - 10.3395i, m_5 = 103.034 + 10.3395i$	91.8642	91.8642%
100	$m_1 = 91.8693, m_2 = 103, m_3 = 0.999998,$ $m_4 = 103.006 - 10.111i, m_5 = 103.006 + 10.111i$	91.8693	91.8693%
1000	$m_1 = 91.872, m_3 = 1.0, m_4 = 103 + 5.08397 \times 10^{-5} i,$ $m_4 = 102.991 - 9.98668i, m_5 = 102.991 + 9.98668i, m_6 = 103$	91.872	91.87%

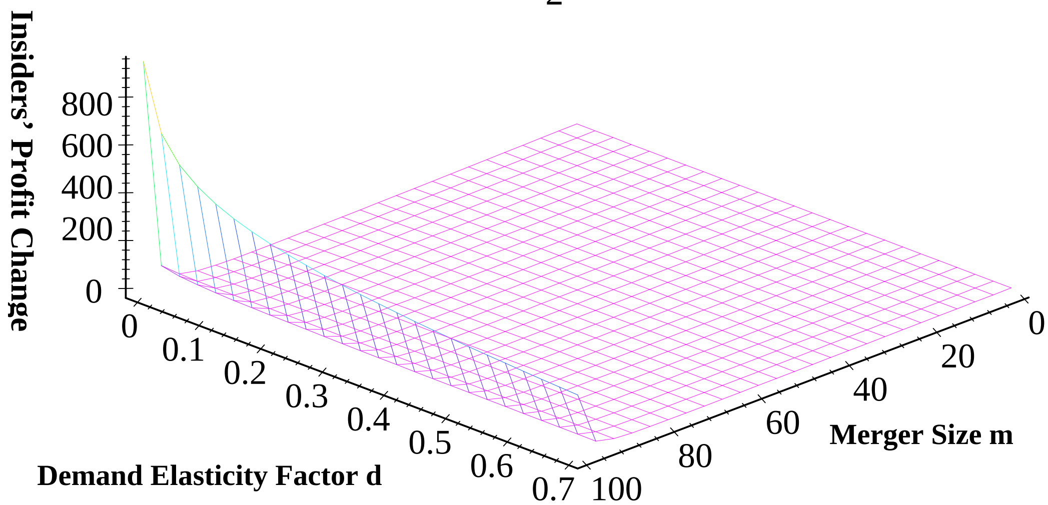
Where:

- n is the market size.
- m values are all the critical merger sizes to let the insiders' pre-merger profit π_M^0 and post-merger profit π_M^* be equal.
- m^*/n is the critical insiders' combined pre-merger market share for a profitable merger.

Table 4.3 gives us an unambiguously increasing trend of the critical merger sizes when the demand elasticity factor is increasingly large. Noted that when d is close to "0", the change in merger size in response to demand elasticity is relatively obvious, thus, we would like to derive the relationship between the profit, demand elasticity and merger size by concentrating this range of d ($-0.005 \le d \le 0.7$). Figure 4.4 illustrates the relationship among them.

Figure 4.4: Insiders' Profit Change in Responds to the Change in Demand Elasticity and Merger Sizes

$$(n=100\ ,\ P=100-Q-\frac{d}{2}Q^2,\ c=40\ ,\ -0.005\le d\le 0.7)$$



As shown In Figure 4.4, an increase in demand elasticity factor d increases the market share required for a merger to be profit enhancing, and the increase in d reduces the profit for a profitable merger. The effect of the demand elasticity on the unprofitable merger could be ignored. No matter what the range d lies in, if the merger size is not large enough, the coalition will never be profitable.

CHAPTER 5

CONCLUSION AND FUTURE WORK

This thesis concentrates on how demand elasticity or the second order derivatives of demand has an effect on the merger's profitability and the critical market share required for a profit enhancing merger in a Cournot oligopoly model with linear cost and quadratic demand. In the class of oligopolies in which P/Q remains unchanged in response to small changes in d, we have obtained a number of new results.

The main contribution of our analysis is to provide the sufficient conditions for a merger to be profitable, and to show how demand elasticity affects the merger's profitability. By studying the pattern of trends in a market consisting of 100 firms, we found that when the demand function is concave, it requires more than 80% combined pre-merger market share to be profitable compared with the SSR model, which predicts that 80% market share is enough for a merger to be profit enhancing. The greater the demand elasticity factor d, the larger the market share that is required for a profitable merger. On the contrary, when the demand function is convex, a merger with less than 80% of the market share would still be profitable.

It is also worthwhile to note that demand elasticity is more influential on the merger's profitability when the demand is relatively elastic, and an increase in the demand elasticity makes producers more profitable.

In our discussion on the welfare effect, we find, in general, social welfare deteriorates after a merger as long as firms are operating at a constant marginal and average cost. However, a merger will be less harmful when the demand is very inelastic (d is a positive large number).

Finally, we would like to refer to some possible extensions of the model as the concluding remarks:

- One main limitation of our analysis is that we could not explore the effect of demand elasticity on the merger's profitability and critical profitable merger sizes when the demand is relatively more elastic. This is due to the restriction we impose on the model $d > -\frac{1}{2a}$, which makes d range from a very small negative number to zero when the demand function is convex. As discussed in chapter 4, we would predict that when d is a very large negative number, or alternatively, the demand is perfectly elastic, much less market share is required for a profitable merger even with constant return to scale, which would be interesting. We could consider an alternative model, such as $P(Q) = a - Q - \frac{d}{1000}Q^2$. This model would allow us to go further in our analysis for the convex demand function case, because it relaxes the restriction to be $d > -\frac{250}{a}$.
- In our analysis, the merger's output is always decreasing compared with their pre-merger level. However, the result could be ambiguous when the merger experiences economies of scale after they form a coalition, which is always true in real life. Firms will expand their output level when the cost is reduced. Thus, we could introduce a scale economy factor into our cost function. Our model could be further expanded when the cost function is asymmetric.

- Finally, applying this model in a heterogeneous goods market would serve as a more challenging analysis, but also one that is of considerable economic interest.

Appendix

Proof of Proposition 1:

For each individual firm i to maximize its profit:

$$\pi_i = (a - Q - \frac{d}{2}Q^2)q_i - cq_i \text{, where } Q = \sum_{k=1}^{n} q_k = \sum_{k \neq i} q_k + q_i$$

or $$\pi_i = \left[a - (\sum_{k \neq i} q_k + q_i) - \frac{d}{2}(\sum_{k \neq i} q_k + q_i)^2 \right] q_i - cq_i$$

The first order condition is:

$$\frac{\partial \pi_i}{\partial q_i} = a - 2q_i - \sum_{k \neq i} q_k - \frac{d}{2}(\sum_{k \neq i} q_k + q_i)^2 - d(\sum_{k \neq i} q_k + q_i)q_i - c = 0 \text{ ,} \qquad \textbf{(A3.1)}$$

or $$a - q_i - Q - \frac{d}{2}Q^2 - dQq_i - c = 0 \qquad \textbf{(A3.1a)}$$

In a symmetric Nash equilibrium, the output of each firm in the industry will be identical, so that, for all i $(i \in M)$ and j $(j \notin M)$, $q_i = q_j = q^0$,

or $$a - q^0 - nq^0 - \frac{d}{2}n^2 q^{0\,2} - dnq^{0\,2} - c = 0 \qquad \textbf{(A3.1b)}$$

Solving for q^0 in equation A3.1b, we get proposition 1.

Check the second order condition: $\frac{\partial^2 \pi_i}{\partial {q_i}^2} = -2 - 2d(\sum_{k \neq i} q_k + q_i) - dq_i < 0$

$\Rightarrow 0 > d > \frac{-2}{(2n+1)q^0}$, or $d > 0$.

Thus, in pre-merger equilibrium,

For each outsider and insider, the individual profit is:

$$\pi_i^0 = (P^0 - c)q_i^0$$

$$= 2\frac{\left[d(a-c)(n+2)+(n+1)\right]\sqrt{(n+1)^2+2nd(a-c)(n+2)}-\left[d(n+2)(2n+1)(a-c)+(n+1)^2\right]}{d^2(n+2)^3 n}$$

For the merged firm M, the combined pre-merger profit is:

$$\pi_M^0 = m(P^0-c)q_i^0$$

$$= 2m\frac{\left[d(a-c)(n+2)+(n+1)\right]\sqrt{(n+1)^2+2nd(a-c)(n+2)}-\left[d(n+2)(2n+1)(a-c)+(n+1)^2\right]}{d^2(n+2)^3 n}$$

The total producer surplus will be:

$$\pi^0 = (P^0-c)Q^0$$

$$= 2\frac{\left[d(a-c)(n+2)+(n+1)\right]\sqrt{(n+1)^2+2nd(a-c)(n+2)}-\left[d(n+2)(2n+1)(a-c)+(n+1)^2\right]}{d^2(n+2)^3}$$

The total consumer surplus will be:

$$CS^0 = \int_0^{Q^0}(a-Q-\frac{d}{2}Q^2)dQ - P^0\times Q^0$$

$$= -\frac{4dn(n+2)(a-c)(n+1-\sqrt{(n+1)^2+2nd(a-c)(n+2)})+(n-2)\left(n+1-\sqrt{(n+1)^2+2nd(a-c)(n+2)}\right)^2}{6d^2(n+2)^3}$$

Q.E.D.

Proof of Proposition 2

(1) The difference between the Cournot pre-merger equilibrium individual firm's output in the quadratic demand model and the linear demand model is:

$$q_i^0 - q_{iL}^0 = \frac{-(n+1)^2+\sqrt{(n+1)^2+2nd\omega(n+2)}(n+1)-nd\omega(n+2)}{dn(n+2)(n+1)} \qquad \textbf{(A3.2.1)}$$

Let the numerator $-(n+1)^2+\sqrt{(n+1)^2+2nd\omega(n+2)}(n+1)-nd\omega(n+2)=\rho_1$

Then $\lim_{d\to 0}\rho_1 = 0$

$$\frac{\partial \rho_1}{\partial d} = \frac{1}{2}\left\{(n+1)^2 + 2nd\omega(n+2)\right\}^{-\frac{1}{2}} \times 2n\omega(n+2)(n+1) - n\omega(n+2)$$

$$= n\omega(n+2)\left[\left\{(n+1)^2 + 2nd\omega(n+2)\right\}^{-\frac{1}{2}}(n+1) - 1\right]$$

When $d > 0, \left\{(n+1)^2 + 2nd\omega(n+2)\right\}^{-\frac{1}{2}}(n+1) < 1 \Rightarrow \left.\begin{array}{l} \frac{\partial \rho_1}{\partial d} < 0 \\ \lim_{d \to 0} \rho_1 = 0 \end{array}\right\}$

$\Rightarrow \rho_1 < 0$, when $d > 0$.

Since the denominator in equation A3.2.1 is positive when d>0

$\Rightarrow \quad q_i^0 < q_{iL}^0$, when $d > 0$.

We could prove when $d < 0, \quad \rho_1 > 0$, then $q_i^0 > q_{iL}^0$ by using the same method.

Thus, compared with the production in the linear demand model, individual firm produces more output when the demand function is convex, and fewer outputs when the demand function is concave.

Q.E.D.

(2) The difference between the Cournot pre-merger equilibrium market price in the quadratic demand model and the linear demand model is:

$$P^0 - P_L^0 = \frac{(n+1)^2 - \sqrt{(n+1)^2 + 2nd\omega(n+2)}(n+1) + nd\omega(n+2)}{d(n+2)^2(n+1)} \quad \textbf{(A3.2.2)}$$

Since $(n+1)^2 - \sqrt{(n+1)^2 + 2nd\omega(n+2)}(n+1) + nd\omega(n+2) = -\rho_1$, we could get the opposite result to (1). Compared with the market price in the linear demand model, market price is lower when the demand function is convex and higher when the demand function is concave.

Q.E.D.

(3) The difference between the Cournot pre-merger equilibrium individual firm's profit in the quadratic demand model and the linear demand model is:

$$\pi_i^0 - \pi_{iL}^0 = \frac{2\left(d\omega(n+2)(n+1)^2 + (n+1)^3\right)\sqrt{(n+1)^2 + 2nd\omega(n+2)} - 2d\omega(n+2)(2n+1)(n+1)^2 - 2(n+1)^4 - d^2\omega^2 n(n+2)^3}{d^2(n+2)^3 n(n+1)^2}$$

(A3.2.3)

Let the numerator: $2\left(d\omega(n+2)(n+1)^2 + (n+1)^3\right)\sqrt{(n+1)^2 + 2nd\omega(n+2)}$
$- 2d\omega(n+2)(2n+1)(n+1)^2 - 2(n+1)^4 - d^2\omega^2 n(n+2)^3 = \rho_2$

Solve for $\rho_2 > 0$

We get $-4\frac{(n+1)^2}{(n+2)^3\omega} < d < 0$

Thus we conclude:

$\rho_2 < 0$, if $d < -4\frac{(n+1)^2}{(n+2)^3\omega}$ *or* $d > 0$

$\rho_2 > 0$, if $-4\frac{(n+1)^2}{(n+2)^3\omega} < d < 0$

$\rho_2 = 0$, if $d = -4\frac{(n+1)^2}{(n+2)^3\omega}$ *or* $d = 0$

Since the denominator in equation A3.2.3 is always positive, thus,

$$\begin{cases} \text{When } d > 0, \text{ we could get } \rho_2 < 0 \Rightarrow \pi_i^0 < \pi_{iL}^0 \\ \text{When } -4\frac{(n+1)^2}{\omega(n+2)^3} < d < 0, \text{ we could get } \rho_2 > 0 \Rightarrow \pi_i^0 > \pi_{iL}^0 \\ \text{When } d < -4\frac{(n+1)^2}{\omega(n+2)^3}, \text{ we could get } \rho_2 < 0 \Rightarrow \pi_i^0 < \pi_{iL}^0 \\ \text{When } d = -4\frac{(n+1)^2}{(n+2)^3\omega}, \text{ we could get } \rho_2 = 0 \Rightarrow \pi_i^0 = \pi_{iL}^0 \end{cases}$$

A merger generally earns more profit when the demand function is linear, except when the demand elasticity factor ranges from $-4\frac{(n+1)^2}{\omega(n+2)^3}$ to 0.

Q.E.D.

Proof of Proposition 3:

After a merger, for each firm i to maximize its profit:

$$\pi_i = \left[a - (\sum_{k \neq i} q_k + q_i) - \frac{d}{2} (\sum_{k \neq i} q_k + q_i)^2 \right] q_i - c q_i$$

The first order condition is:

$$\frac{\partial \pi_i}{\partial q_i} = a - 2q_i - \sum_{k \neq i} q_k - \frac{d}{2} (\sum_{k \neq i} q_k + q_i)^2 - d (\sum_{k \neq i} q_k + q_i) q_i - c = 0 \qquad \textbf{(A3.3)}$$

Specifically, for the merged firm M to maximize its profit:

$$\pi_M = \left[a - (\sum_{k \neq M} q_k + q_M) - \frac{d}{2} (\sum_{k \neq M} q_k + q_M)^2 \right] q_M - c q_M$$

The first order condition is:

$$\frac{\partial \pi_M}{\partial q_M} = a - 2q_M - \sum_{k \neq M} q_k - \frac{d}{2} (\sum_{k \neq M} q_k + q_M)^2 - d (\sum_{k \neq M} q_k + q_M) q_M - c = 0 \qquad \textbf{(A3.3a)}$$

There are $(n - m + 1)$ firms in the market after merger, then:

for all j $(j \notin M)$ and M $(i \in M)$, $q_j = q_M = q^*$

$$Q = (n - m + 1) q^*$$

or $a - q^* - Q - \frac{d}{2} Q^2 - dQq^* - c = 0$

or $a - q^* - (n - m + 1) q^* - \frac{d}{2} (n - m + 1)^2 q^{*2} - d (n - m + 1) q^* - c = 0$ **(A3.3b)**

Solve for q^* in equation A3.3b leads to Proposition 3.

Check the second order condition:

$$\frac{\partial^2 \pi_M}{\partial q_M{}^2} = -2 - 2d (\sum_{k \neq M} q_k + q_M) - dq_M < 0$$

$$\Rightarrow 0 > d > \frac{-2}{\{2(n - m + 1) + 1\} q^*}$$

To simplify the notations, let $n^* = n - m + 1$

Thus, for each outsider and the coalition, the profit will be:

$$\pi_j^* = \pi_M^* = (P^* - c) q^*$$

$$= 2\frac{\left[d(a-c)(n^*+2)+(n^*+1)\right]\sqrt{(n^*+1)^2+2d(a-c)n^*(n^*+2)}-\left[d(n^*+2)(2n^*+1)(a-c)+(n^*+1)^2\right]}{d^2(n^*+2)^3 n^*}$$

The producer surplus after merger will be:

$$\pi^* = (P^*-c)Q^*$$

$$= 2\frac{\left[d(a-c)(n^*+2)+(n^*+1)\right]\sqrt{(n^*+1)^2+2d(a-c)n^*(n^*+2)}-\left[d(n^*+2)(2n^*+1)(a-c)+(n^*+1)^2\right]}{d^2(n^*+2)^3}$$

The consumer surplus will be:

$$CS^* = \int_0^{Q^*}\left(a-Q-\frac{d}{2}Q^2\right)dQ - P^*\times Q^* = aQ^* - \frac{1}{2}Q^{*2} - \frac{d}{6}Q^{*3} - P^*\times Q^*$$

$$= -\frac{4dn^*(n^*+2)(a-c)(n^*+1-\sqrt{(n^*+1)^2+2d(a-c)n^*(n^*+2)})+(n^*-2)\left(n^*+1-\sqrt{(n^*+1)^2+2d(a-c)n^*(n^*+2)}\right)^2}{6d^2(n^*+2)^3}$$

Q.E.D.

Proof of Proposition 4 (When d>0)

To simplify the notations,

Let $\omega = a-c$

$X = n+1$

$Y = n-m+2$

(1) The difference between a coalition's post-merger output and combined pre-merger output is:

$$q^* - mq^0 = \frac{\left(-Y+\sqrt{Y^2+2d\omega(Y^2-1)}\right)(X^2-1) - m\left(-X+\sqrt{X^2+2d\omega(X^2-1)}\right)(Y^2-1)}{d(X^2-1)(Y^2-1)} \qquad \textbf{(A3.4.1)}$$

Let the numerator $\left(-Y+\sqrt{Y^2+2d\omega(Y^2-1)}\right)(X^2-1) - m\left(-X+\sqrt{X^2+2d\omega(X^2-1)}\right)(Y^2-1) = \varphi_1$

Then $\lim_{d\to 0}\varphi_1 = 0$

$$\frac{\partial\varphi_1}{\partial d}=\frac{1}{2}\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}\times 2\omega(Y^2-1)(X^2-1)-\frac{m}{2}\left\{X^2+2d\omega(X^2-1)\right\}^{-\frac{1}{2}}\times 2\omega(X^2-1)(Y^2-1)$$

$$=(X^2-1)(Y^2-1)\omega\left[\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}^{-\frac{1}{2}}\right] \qquad \textbf{(A3.4.1a)}$$

Let $\varphi_{11}=\left\{Y^2+2d\omega(Y^2-1)\right\}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}$ from equation A3.4.1a

$$=(Y^2-\frac{X^2}{m^2})+2d\omega\left\{\frac{m^2(Y^2-1)-(X^2-1)}{m^2}\right\}$$

Since:

- $Y^2-\frac{X^2}{m^2}=(Y+\frac{X}{m})(Y-\frac{X}{m})=(Y+\frac{X}{m})\left[(n-m+2)-\frac{n+1}{m}\right]$

  $=(Y+\frac{X}{m})\left(\frac{(m-1)(n-m+1)}{m}\right)>0$

- $m^2(Y^2-1)-(X^2-1)=m^2\left\{(n-m+2)^2-1\right\}-\left\{(n+1)^2-1\right\}$

  $=m^2n^2-2m^3n+4m^2n+m^4-4m^3+3m^2-n^2-2n$

  $=(m-1)\left\{(m-1)(m-n)^2+2m(n-m)+2n(n-m+1)\right\}>0$

$\Rightarrow \varphi_{11}>0$, or $Y^2+2d\omega(Y^2-1)>\frac{X^2+2d\omega(X^2-1)}{m^2}$

Since when $f(x)=x^{-\frac{1}{2}}$, $f(x)\uparrow$ *with* $x\downarrow$

$$\Rightarrow\left[\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}^{-\frac{1}{2}}\right]<0\Rightarrow\frac{\partial\varphi_1}{\partial d}<0$$

$\left\{\begin{array}{l}\frac{\partial\varphi_1}{\partial d}<0\\ \lim_{d\to 0}\varphi_1=0\end{array}\right\}\Rightarrow$ When $d>0,\ \ \varphi_1<0$,

Since the denominator for equation 3.4.1 is positive,

$\Rightarrow\ \ q^*<mq^0$

The coalition's total output contracts in relative to its pre-merger level.

Q.E.D.

(2) The individual outsider's output change is given by

$$q^* - q^0 = \frac{\left(-Y + \sqrt{Y^2 + 2d\omega(Y^2-1)}\right)(X^2-1) - \left(-X + \sqrt{X^2 + 2d\omega(X^2-1)}\right)(Y^2-1)}{d(X^2-1)(Y^2-1)} \quad \textbf{(A3.4.2)}$$

Let the numerator $\left(-Y + \sqrt{Y^2 + 2d\omega(Y^2-1)}\right)(X^2-1) - \left(-X + \sqrt{X^2 + 2d\omega(X^2-1)}\right)(Y^2-1) = \varphi_2$

Then $\lim_{d \to 0} \varphi_2 = 0$

$$\frac{\partial \varphi_2}{\partial d} = (X^2-1)(Y^2-1)\omega\left[\left\{Y^2 + 2d\omega(Y^2-1)\right\}^{-\frac{1}{2}} - \left\{X^2 + 2d\omega(X^2-1)\right\}^{-\frac{1}{2}}\right]$$

$$\left.\begin{array}{l}\text{Since } \left\{Y^2 + 2d\omega(Y^2-1)\right\} < \left\{X^2 + 2d\omega(X^2-1)\right\}, \\ \text{and for function } f(x) = x^{-\frac{1}{2}}, \quad f(x)\downarrow \ \ with \ \ x\uparrow \end{array}\right\}$$

$$\left.\begin{array}{l}\Rightarrow \left\{Y^2 + 2d\omega(Y^2-1)\right\}^{-\frac{1}{2}} > \left\{X^2 + 2d\omega(X^2-1)\right\}^{-\frac{1}{2}} \\ (X^2-1)(Y^2-1)\omega > 0 \end{array}\right\}$$

$$\Rightarrow \frac{\partial \varphi_2}{\partial d} > 0$$

$$\left.\begin{array}{l}\left\{\begin{array}{l}\frac{\partial \varphi_2}{\partial d} > 0 \\ \lim_{d \to .0} \varphi_1 = 0\end{array}\right\} \Rightarrow \text{ When } \ d > 0, \ \ \varphi_2 > 0\,, \\ \text{Since the denominator for equation A3.4.2 is positive,}\end{array}\right\}$$

$$\Rightarrow \quad q^* > q^0$$

Each outsider will expand individual output following the merger.

Q.E.D.

(3) The total output change in the market is:

$$Q^* - Q^0 = \frac{-(Y+1)\left\{-X + \sqrt{X^2 + 2d\omega(X^2-1)}\right\} + (X+1)\left\{-Y + \sqrt{Y^2 + 2d\omega(Y^2-1)}\right\}}{d(X+1)(Y+1)} \quad \textbf{(A3.4.3)}$$

The unique Cournot-Nash Equilibrium (CNE) is characterized by the F.O.C conditions for profit maximization of n firms:

$$P(Q) + \frac{\partial P(Q)}{\partial Q} q_i - c = 0 \quad \textbf{(A3.4.3a)}$$

$$\left.\begin{array}{l} \left.\begin{array}{l} P(Q^0)+\dfrac{\partial P(Q^0)}{\partial Q^0}q_i(Q^0)-c=0 \\ q_i(Q^*)>q_i(Q^0) \\ \dfrac{\partial P(Q^0)}{\partial Q^0}<0 \end{array}\right\} \Rightarrow P(Q^0)+\dfrac{\partial P(Q^0)}{\partial Q^0}q_i(Q^*)-c<0 \\ \qquad\qquad\qquad\qquad\qquad\qquad P(Q^*)+\dfrac{\partial P(Q^*)}{\partial Q^*}q_i(Q^*)-c=0 \\ \dfrac{\partial P(Q)}{\partial Q}+\dfrac{\partial^2 P(Q)}{\partial Q^2}q_i=-1-d(Q+q)<0\text{, then equation (A3.4.3a) is decreasing in Q.} \end{array}\right\}$$

$$\Rightarrow Q^* < Q^0$$

The Merger leads to the contraction of the total output in the market relative to the pre-merger level.

Q.E.D.

(4) Price will rise after merging.

$$P^* - P^0 = (Q^0 - Q^*)\left\{1+\frac{d}{2}(Q^* + Q^0)\right\} > 0 \qquad \textbf{(A3.4.4)}$$

Q.E.D.

Proof of Proposition 4 (When d<0)

(1) The difference between a coalition's post-merger output and combined pre-merger output is:

$$q^* - mq^0 = \frac{\left(-Y+\sqrt{Y^2+2d\omega(Y^2-1)}\right)(X^2-1)-m\left(-X+\sqrt{X^2+2d\omega(X^2-1)}\right)(Y^2-1)}{d(X^2-1)(Y^2-1)} \qquad \textbf{(A3.4.1)}$$

Let the numerator $\left(-Y+\sqrt{Y^2+2d\omega(Y^2-1)}\right)(X^2-1)-m\left(-X+\sqrt{X^2+2d\omega(X^2-1)}\right)(Y^2-1)=\delta_1$

Then $\lim_{d\to 0}\delta_1 = 0$

$$\frac{\partial\delta_1}{\partial d}=\frac{1}{2}\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}\times 2\omega(Y^2-1)(X^2-1)-\frac{m}{2}\left\{X^2+2d\omega(X^2-1)\right\}^{-\frac{1}{2}}\times 2\omega(X^2-1)(Y^2-1)$$

$$=(X^2-1)(Y^2-1)\omega\left[\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}^{-\frac{1}{2}}\right]$$

$$\left\{Y^2+2d\omega(Y^2-1)\right\}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}$$

$$=(Y^2-\frac{X^2}{m^2})+2d\omega\left\{\frac{m^2(Y^2-1)-(X^2-1)}{m^2}\right\}$$

$$>(Y^2-\frac{X^2}{m^2})-\frac{\omega}{a}\left\{\frac{m^2(Y^2-1)-(X^2-1)}{m^2}\right\}, \text{ since } \quad d>-\frac{1}{2a}$$

$$=\frac{\omega}{a}(1-\frac{1}{m^2})+\frac{c}{a}(Y^2-\frac{X^2}{m^2})>0$$

$$\Rightarrow \left\{Y^2+2d\omega(Y^2-1)\right\}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}>0$$

Since for function $f(x)=x^{-\frac{1}{2}}$, $f(x)\uparrow$ *with* $x\downarrow$

$$\Rightarrow\left[\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}-\left\{\frac{X^2+2d\omega(X^2-1)}{m^2}\right\}^{-\frac{1}{2}}\right]<0\Rightarrow\frac{\partial\delta_1}{\partial d}<0$$

$\left\{\begin{array}{l}\frac{\partial\delta_1}{\partial d}<0\\ \lim\limits_{d\to 0}\delta_1=0\end{array}\right\}\Rightarrow$ When $d<0$, $\delta_1>0$,

Since the denominator for equation A3.4.1 is negative,

$\Rightarrow \quad q^*<mq^0$, which is the same result as when d>0.

Q.E.D.

(2) The individual outsider's output change is given by:

$$q^*-q^0=\frac{\left(-Y+\sqrt{Y^2+2d\omega(Y^2-1)}\right)(X^2-1)-\left(-X+\sqrt{X^2+2d\omega(X^2-1)}\right)(Y^2-1)}{d(X^2-1)(Y^2-1)} \qquad \textbf{(A3.4.2)}$$

Let the numerator $\left(-Y+\sqrt{Y^2+2d\omega(Y^2-1)}\right)(X^2-1)-\left(-X+\sqrt{X^2+2d\omega(X^2-1)}\right)(Y^2-1)=\delta_2$

Then $\lim\limits_{d\to 0}\delta_2=0$

$$\frac{\partial\delta_2}{\partial d}=(X^2-1)(Y^2-1)\omega\left[\left\{Y^2+2d\omega(Y^2-1)\right\}^{-\frac{1}{2}}-\left\{X^2+2d\omega(X^2-1)\right\}^{-\frac{1}{2}}\right]$$

Since $\left\{Y^2+2d\omega(Y^2-1)\right\}-\left\{X^2+2d\omega(X^2-1)\right\}$

$$= (Y^2 - X^2)(1 + 2d\omega)$$

From lemma 1, we know $2d > -\frac{1}{a}$

$$\left.\begin{array}{l} \Rightarrow (1+2d\omega) > (1-\frac{\omega}{a}) = \frac{c}{a} > 0 \\ \text{Since } \ (Y^2 - X^2) < 0 \end{array}\right\}$$

$$\left.\begin{array}{l} \Rightarrow \{Y^2 + 2d\omega(Y^2-1)\} < \{X^2 + 2d\omega(X^2-1)\} \\ \text{Since for function } \ f(x) = x^{-\frac{1}{2}}, \ \ f(x)\downarrow \ \ with \ \ x\uparrow \end{array}\right\}$$

$$\Rightarrow \{Y^2 + 2d\omega(Y^2-1)\}^{-\frac{1}{2}} > \{X^2 + 2d\omega(X^2-1)\}^{-\frac{1}{2}} \Rightarrow \frac{\partial \delta_2}{\partial d} > 0$$

$$\left.\begin{array}{l} \left\{\begin{array}{l} \frac{\partial \delta_2}{\partial d} > 0 \\ \lim\limits_{d \to .0} \delta_2 = 0 \end{array}\right\} \Rightarrow \text{ When } \ d < 0, \ \ \delta_2 < 0 \\ \text{Since the denominator of equation A3.4.2 is negative,} \end{array}\right\}$$

$\Rightarrow \quad q^* > q^0$, which is the same result as when d>0.

Q.E.D.

(3) The total output change in the market is:

$$Q^* - Q^0 = \frac{-(Y+1)\left\{-X + \sqrt{X^2 + 2d\omega(X^2-1)}\right\} + (X+1)\left\{-Y + \sqrt{Y^2 + 2d\omega(Y^2-1)}\right\}}{d(X+1)(Y+1)} \qquad \textbf{(3.4.3)}$$

Proof: $P(Q) + \frac{\partial P(Q)}{\partial Q} q_i - c = 0$ **(3.4.3a)**

$$\left.\begin{array}{l} \left.\begin{array}{l} P(Q^0) + \frac{\partial P(Q^0)}{\partial Q^0} q_i(Q^0) - c = 0 \\ q_i(Q^*) > q_i(Q^0) \\ \frac{\partial P(Q^0)}{\partial Q^0} < 0 \end{array}\right\} \Rightarrow P(Q^0) + \frac{\partial P(Q^0)}{\partial Q^0} q_i(Q^*) - c < 0 \\ \qquad P(Q^*) + \frac{\partial P(Q^*)}{\partial Q^*} q_i(Q^*) - c = 0 \\ \frac{\partial P(Q)}{\partial Q} + \frac{\partial^2 P(Q)}{\partial Q^2} q_i = -1 - d(Q+q) < 0 \ \text{ from A1, then equation (A3.4.3a) is decreasing in Q.} \end{array}\right\}$$

$\Rightarrow Q^0 > Q^*$, which is the same result as when $d > 0$.

Q.E.D.

(4) Price will rise after merging.

$$P^* - P^0 = (Q^0 - Q^*)\left\{1 + \frac{d}{2}(Q^* + Q^0)\right\}$$

Since $\frac{d}{2}(Q^* + Q^0) > -\frac{1}{\{2(n-m+1)+1\}q_i^*}Q^* - \frac{1}{(2n+1)q_i^0}Q^0$, because of second order conditions:

- $d > \frac{-2}{(2n+1)q_i^0}$, from proposition 1.
- $d > \frac{-2}{\{2(n-m+1)+1\}q_i^*}$, from proposition 3

$$\left.\begin{array}{r} \Rightarrow \left\{1 + \frac{d}{2}(Q^* + Q^0)\right\} > 1 - \frac{1}{\{2(n-m+1)+1\}q_i^*}(n-m+1)q_i^* - \frac{1}{(2n+1)q_i^0}nq_i^0 = \frac{2n-m+2}{(2n+1)(2n-2m+3)} > 0 \\ Q^* < Q^0 \text{, from Proposition 4.(3)} \end{array}\right\}$$

$\Rightarrow P^* - P^0 > 0$

Q.E.D.

Table A1: The Critical Merger Sizes and Market Shares for Profitable Mergers Given Different Demand Elasticity Factors and Market Sizes[10]

Where:

- n is the market size
- m values are all the critical merger sizes to allow the insiders' combined pre-merger profit π_M^0 and post-merger profit π_M^* to be equal.
- m^* is the feasible critical merger size for a profitable merger.
- m^*/n is the least insiders' combined pre-merger market share for a profitable merger.

d	n	m values	m^*	m*/n
-0.005	**3**	$m_1 = 2.36655, m_2 = 1.0,$ $m_3 = 7.91203 - 5.20187i, m_4 = 7.91203 + 5.20187i$	2.36655	78.89%
	5	$m_1 = 3.89158, m_2 = 1.0,$ $m_3 = 10.415 - 6.89765i, m_4 = 10.416 + 6.89765i$	3.89158	77.83%
	6	$m_1 = 4.68648, m_2 = 1.0,$ $m_3 = 11.6322 - 7.66484i, m_4 = 116.6322 + 7.66484i$	4.68648	78.11%
	7	$m_1 = 5.49654, m_2 = 1.0,$ $m_3 = 12.83 - 8.3909i, m_4 = 12.83 + 8.3909i$	5.49654	78.52%
	8	$m_1 = 6.31887, m_2 = 1.0,$ $m_3 = 14.0124 - 9.08225i, m_4 = 14.0124 + 9.08225i$	6.31887	78.99%
	10	$m_1 = 7.9925, m_2 = 1.0,$ $m_3 = 16.3392 - 0.3793i, m_4 = 16.3392 + 10.3793i$	7.9925	79.93%
	20	$m_1 = 16.7003, m_2 = 1.0,$ $m_3 = 27.5097 - 15.7726i, m_4 = 27.5097 + 15.7726i$	16.7003	83.50%
	30	$m_1 = 25.7026, m_2 = 1.0,$ $m_3 = 38.2725 - 20.1252i, m_4 = 38.2725 + 20.1252i$	25.7026	85.68%
	40	$m_1 = 34.8596, m_2 = 1.0,$ $m_3 = 48.8282 - 23.8812i, m_4 = 48.8282 + 23.8812i$	34.8596	87.15%
	50	$m_1 = 44.1159, m_2 = 1.0,$ $m_3 = 59.2577 - 27.2336i, m_4 = 59.2577 + 27.2336i$	44.1159	88.23%

[10] When $d \neq 0$,

$$\pi_M^* - \pi_M^0 = 2\frac{\{d\omega(Y+1)(-2Y+\sqrt{\beta}+1)+Y(-Y+\sqrt{\beta})\}}{d^2(Y+1)^2(Y^2-1)} - 2m\frac{\{d\omega(X+1)(-2X+\sqrt{\alpha}+1)+X(-X+\sqrt{\alpha})\}}{d^2(X+1)^2(X^2-1)}$$

When d=0, $\pi_M^* - \pi_M^0 = \frac{(a-c)^2}{(n-m+2)^2} - m\frac{(a-c)^2}{(n+1)^2}$

	60	$m_1 = 53.4431, m_2 = 1.0,$ $m_3 = 69.6026 - 30.2885i, m_4 = 69.6026 + 30.2885i$	53.4431	89.07%
	70	$m_1 = 62.8241, m_2 = 1.0,$ $m_3 = 79.8873 - 33.1119i, m_4 = 79.8873 + 33.1119i$	62.8241	89.75%
	80	$m_1 = 72.2477, m_2 = 1.0,$ $m_3 = 90.1271 - 35.7483i, m_5 = 90.1271 + 35.7483i$	72.2477	90.31%
	90	$m_1 = 81.7062, m_2 = 1.0,$ $m_3 = 100.332 - 38.2297i, m_4 = 100.332 + 38.2297i$	81.7062	90.785%
	100	$m_1 = 91.1939, m_2 = 1.0,$ $m_3 = 110.51 - 40.5798i, m_4 = 110.51 + 40.5798i$	91.1939	91.19%
d	**n**	**m values**	**m***	**m*/n**
0	**3**	$m_1 = 2.43845, m_2 = 1.0, m_3 = 6.56155$	2.43845	81.28%
	5	$m_1 = 4.0, m_2 = 1.0, m_3 = 9$	4	80%
	6	$m_1 = 4.80742, m_2 = 1.0, m_3 = 10.1926$	4.80742	80.12%
	7	$m_1 = 5.62772, m_2 = 1.0, m_3 = 11.3723$	5.62772	80.40%
	8	$m_1 = 6.45862, m_2 = 1.0, m_3 = 12.5414$	6.45862	80.73%
	10	$m_1 = 8.1459, m_2 = 0.999999, m_3 = 14.8541$	8.1459	81.46%
	20	$m_1 = 16.8902, m_2 = 0.999999, m_3 = 16.8902$	16.8902	84.45%
	30	$m_1 = 25.9098, m_2 = 0.999999, m_3 = 37.0902$	25.9098	86.37%
	40	$m_1 = 35.0774, m_2 = 1.0, m_3 = 47.9226$	35.0774	87.69%
	50	$m_1 = 44.3411, m_2 = 1.0, m_3 = 58.6589$	44.3411	88.68%
	60	$m_1 = 53.6738, m_2 = 1.0, m_3 = 69.3262$	53.6738	89.46%
	70	$m_1 = 63.059, m_2 = 1.0, m_3 = 79.941$	63.059	90.08%
	80	$m_1 = 72.4861, m_2 = 1.0, m_3 = 90.5139$	72.4861	90.61%
	90	$m_1 = 81.9475, m_2 = 1.0, m_3 = 101.052$	81.9475	91.05%
	100	$m_1 = 91.4377, m_2 = 0.999999, m_3 = 111.562$	91.4377	91.44%
d	**n**	**m values**	**m***	**m*/n**
0.05	**3**	$m_1 = 2.53012, m_2 = 0.999998,$ $m_3 = 6.54682 - 3.27804i, m_4 = 6.54682 + 3.27804i$	2.53012	84.34%
	5	$m_1 = 4.13216, m_2 = 0.999999,$ $m_3 = 8.73164 - 4.2549i, m_4 = 8.73164 + 4.2549i$	4.13216	82.64%
	6	$m_1 = 4.95299, m_2 = 1.0,$ $m_3 = 9.80577 - 4.68696i, m_4 = 9.80577 + 4.68696i$	4.95299	82.55%
	7	$m_1 = 5.78412, m_2 = 1.0,$ $m_3 = 10.8712 - 5.09188i, m_4 = 10.8712 + 5.09188i$	5.78412	82.63%
	8	$m_1 = 6.62399, m_2 = 1.0,$ $m_3 = 11.9295 - 5.47436i, m_4 = 11.9295 + 5.47436i$	6.62399	82.80%

	10	$m_1 = 8.32541, m_2 = 1.0,$ $m_3 = 14.0295 - 6.18497i, m_4 = 14.0295 + 6.18497i$	8.32541	83.25%
	20	$m_1 = 17.1067, m_2 = 1.0,$ $m_3 = 24.3438 - 9.06484i, m_4 = 24.3438 + 9.06484i$	17.1067	85.53%
	30	$m_1 = 26.1434, m_2 = 0.999998,$ $m_3 = 34.5146 - 11.3237i, m_4 = 34.5146 + 11.3237i$	26.1434	87.14%
	40	$m_1 = 35.3212, m_2 = 1.0,$ $m_3 = 44.6237 - 13.2399i, m_4 = 44.6237 + 13.239i$	35.3212	88.30%
	50	$m_1 = 44.592, m_2 = 0.999999,$ $m_3 = 54.7 - 14.9309i, m_4 = 54.7 + 14.9309i$	44.592	89.18%
	60	$m_1 = 53.9299, m_2 = 1.0,$ $m_3 = 64.7564 - 16.4597i, m_4 = 64.7564 + 16.4597i$	53.9299	89.88%
	70	$m_1 = 63.3193, m_2 = 0.999999,$ $m_3 = 74.8 - 17.8648i, m_4 = 74.8 + 17.8648i$	63.3193	90.46%
	80	$m_1 = 72.7497, m_2 = 1.0,$ $m_3 = 84.8346 - 19.1714i, m_4 = 84.8346 + 19.1714i$	72.7497	90.94%
	90	$m_1 = 82.2138, m_2 = 1.0,$ $m_3 = 94.8629 - 20.3975i, m_4 = 94.8629 + 20.3975$	82.2138	91.35%
	100	$m_1 = 91.7063, m_2 = 1.0,$ $m_3 = 104.886 - 21.5561i, m_4 = 104.886 + 21.5561i$	91.7063	91.71%
d	**n**	**m values**	**m***	**m*/n**
5	**3**	$m_1 = 2.58363, m_2 = 0.999997,$ $m_3 = 5.88364 + 1.74027i, m_4 = 5.88364 - 1.74027i$	2.58363	86.12%
	5	$m_1 = 4.20868, m_2 = 0.999996,$ $m_3 = 7.91895 + 2.26682i, m_4 = 7.91895 - 2.26682i$	4.20868	84.17%
	6	$m_1 = 5.03702, m_2 = 1.0,$ $m_3 = 8.933 + 2.49445i, m_4 = 8.933 - 2.49445i$	5.03702	83.95%
	7	$m_1 = 5.87417, m_2 = 0.999999,$ $m_3 = 9.94516 + 2.70547i, m_4 = 9.94516 - 2.70547i$	5.87417	83.92%
	8	$m_1 = 6.71901, m_2 = 1.0,$ $m_3 = 10.9558 + 2.90304i, m_4 = 10.9558 - 2.90304i$	6.71901	83.99%
	10	$m_1 = 8.42815, m_2 = 1.0,$ $m_3 = 12.9734 + 3.26621i, m_4 = 12.9734 - 3.26621i$	8.42815	84.28%
	20	$m_1 = 17.2292, m_2 = 0.999999,$ $m_3 = 23.0228 + 4.70244i, m_4 = 23.0228 - 4.70344i$	17.2292	86.15%
	30	$m_1 = 26.2747, m_2 = 1.0,$ $m_3 = 33.0458 + 5.80922i, m_4 = 33.0458 - 5.80922i$	26.2747	87.58%
	40	$m_1 = 35.4581, m_2 = 0.999999, m_3 = 43.0591 + 6.7407i,$ $m_4 = 43.0591 - 6.7407i, m_5 = 40.9967$	35.4581	88.65%

	50	$m_1 = 44.7324, m_2 = 0.999996, m_3 = 53.0677 + 7.56002i,$ $m_4 = 53.0677 - 7.56002i, m_5 = 53.0677 - 7.56002i$	44.7324	89.46%
	60	$m_1 = 54.0729, m_2 = 0.999997, m_3 = 63.0739 + 8.29985i,$ $m_4 = 63.0739 - 8.29985i, m_5 = 60.9967$	54.0729	90.12%
	70	$m_1 = 63.4642, m_2 = 1.0, m_3 = 73.0784 + 8.97951i,$ $m_4 = 73.0784 - 8.97951i, m_5 = 70.9967$	63.4642	90.66%
	80	$m_1 = 72.8962, m_2 = 0.999999, m_3 = 83.0819 + 9.61155i,$ $m_4 = 83.0819 - 9.61155i, m_5 = 80.9967$	72.8962	91.12%
	90	$m_1 = 82.3617, m_2 = 1.0, m_3 = 93.0847 + 10.2048i,$ $m_4 = 93.0847 - 10.2048i, m_5 = 90.9967$	82.3617	91.51%
	100	$m_1 = 91.8553, m_2 = 1.0, m_3 = 103.087 + 10.7655i,$ $m_4 = 103.087 - 10.7655i, m_5 = 100.997$	91.8553	91.86%
d	**n**	**m values**	**m***	**m*/n**
1000	**3**	$m_1 = 2.58967, m_2 = 1.00001, m_3 = 5.83508 + 1.61495i,$ $m_4 = 5.83508 - 1.61485i, m_5 = 4.00002, m_6 = 5.99999$	2.58967	86.32%
	5	$m_1 = 4.21737, m_2 = 1.0, m_3 = 7.86148 + 2.10706i,$ $m_4 = 7.86148 - 2.10706i, m_5 = 6.0001, m_6 = 7.99999$	4.21737	84.35%
	6	$m_1 = 5.04664, m_2 = 0.999998, m_3 = 8.87218 + 2.31933i,$ $m_4 = 8.87218 - 2.31933i, m_5 = 6.99987$	5.04664	84.11%
	7	$m_1 = 5.88446, m_2 = 0.999999, m_3 = 9.88154 + 2.51586i,$ $m_4 = 9.88154 - 2.51586i, m_5 = 13$	5.88446	84.06%
	8	$m_1 = 6.72985, m_2 = 1.0,$ $m_3 = 10.8897 + 2.69969i, m_4 = 10.8897 - 2.69969i,$	6.72985	84.12%
	10	$m_1 = 8.4399, m_2 = 0.999998, m_3 = 12.9033 + 3.0373i,$ $m_4 = 12.9033 - 3.0373i, m_5 = 13$	8.4399	84.40%
	20	$m_1 = 17.2431, m_2 = 0.999998, m_3 = 22.9416 + 4.3708i,$ $m_4 = 22.9416 - 4.3708i, m_5 = 21.0004, m_6 = 23$	17.2431	86.22%
	30	$m_1 = 26.2896, m_2 = 1.0, m_3 = 32.9592 + 5.39554i,$ $m_4 = 32.9592 - 5.39554i, m_5 = 31.0004, m_6 = 33$	26.2896	87.63%
	40	$m_1 = 35.4735, m_2 = 1.0, m_3 = 42.9693 + 6.25843i,$ $m_4 = 42.9693 - 6.25843i, m_5 = 40.9983$	35.4735	88.68%
	50	$m_1 = 44.7479, m_2 = 0.999997, m_3 = 52.976 + 7.01711i,$ $m_4 = 52.976 - 7.01711i, m_5 = 51.0021, m_6 = 53$	44.7479	89.50%
	60	$m_1 = 54.0884, m_2 = 0.999998, m_3 = 62.9808 + 7.70215i,$ $m_4 = 62.9808 - 7.70215i, m_5 = 61.0068, m_6 = 63$	54.0884	90.15%
	70	$m_1 = 63.4797, m_2 = 1.0, m_3 = 72.9843 + 8.33145i,$ $m_4 = 72.9843 - 8.33145i, m_5 = 71.0123, m_6 = 73$	63.4797	90.69%

	80	$m_1 = 72.9133, m_2 = 1.0, m_3 = 82.9866 + 8.91828i,$ $m_4 = 82.9866 - 8.91828i, m_5 = 83 + 2.54662 \times 10^{-6} i,$ $m_6 = 83 - 2.54662 \times 10^{-6} i$	72.9133	91.14%
	90	$m_1 = 82.3782, m_2 = 1.0, m_3 = 92.9888 + 9.4672i,$ $m_4 = 92.9888 - 9.4672i, m_5 = 92.9999 - 3.02059 \times 10^{-5} i,$ $m_6 = 93$	82.3782	91.53%
	100	$m_1 = 91.872, m_2 = 1.0, m_3 = 102.991 + 9.98668i,$ $m_4 = 102.991 - 9.98668i, m_5 = 103 + 5.08397 \times 10^{-5} i,$ $m_6 = 103$	91.872	91.87%

BIBLIOGRAPHY

Deneckere, R. and Davidson, C. (1985) “The Incentives to Form Coalitions with Bertrand Competition.” *The RAND Journal of Economics*, Vol. 16, No. 4, pp. 473-486.

Farrell, J., and Shapiro, C. (1990) "Horizontal Mergers: An Equilibrium Analysis." *The American Economic Review* 80, pp. 107-126.

Hahn, F. (1962) “The Stability of the Cournot Oligopoly Solution.” *Review of Economic Studies*, 29, pp. 329-331.

Hay, G. and Werden, G. (1993) “Horizontal Mergers: Law, Policy and Economics.” *The American Economic Review*, Vol. 83, No. 2, pp. 173-177.

Heubeck, S., Smythe, D., and Zhao J. (2005) “The relationship between cost asymmetry and merger efficiency.” Forthcoming in *Annals of Economics & Finance*.

Kimmel, S. (1992) “Effects of Cost Changes on Oligopolists’ Profits.” *The Journal of Industrial Economics*” Vol. 40, No. 4, pp. 441-449.

Kovacic, W. and Shapiro, C. (2000) “Antitrust Policy: A Century of Economic and Legal Thinking” *The Journal of Economic Perspectives*, Vol. 14, No. 1, pp. 43-60.

Laffont, J. (1994) “The New Economics of Regulation Ten Years After.” *Econometrica,* Vol. 62, No.3 pp. 507-537.

Levin, D. (1990) "Horizontal mergers: The 50-percent benchmark." *The American Economic Review* 80(5), pp. 1238-1245.

Mankiw, G. and Whinston, M. (1986) "Free Entry and Social Inefficiency." *The RAND Journal of Economics*, Vol. 17, No. 1, pp. 48-58.

Perry, M. and Porter, R. (1985) "Oligopoly and the Incentive for Horizontal Merger" *The American Economic Review*, Vol. 75, No. 1, pp. 219-227.

Ruffin, R. (1971) "Cournot Oligopoly and Competitive Behavior." *Review of Economic Studies*, 38, pp. 493-502.

Salant, S., Switzer, S., and Reynolds, R. (1983) "Losses from horizontal merger: The effects of an exogenous change in industry structure on cournot-nash equilibrium." *Quarterly Journal of Economics* 98(2), pp. 185-200.